CLASS: IMSc.   SEMESTER : IX
BRANCH: MATHEMATICS AND COMPUTING   SESSION : MO/2024

SUBJECT: MA502 NUMBER THEORY

TIME: 3 Hours   FULL MARKS: 50

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	BL
Q.1(a)	Use Pollard's rho method to find a factor of 1189.	[5]	CO1	
Q.1(b)	State Solovay-Strassen Algorithm.	[5]	CO4	
	Is 2 an Euler witness for 133?			
Q.2(a)	Determine whether	[5]	CO1	
	(i) $x^2 \equiv -35 (mod\, 97)$ and			
	(ii) $x^2 \equiv 101\, (mod\, 1987)$			
	are solvable.			
Q.2(b)	Write down the Euler's criteria in terms of the Legendre's symbol.	[5]	CO4	
	Prove that (-1|p) is 1 if $p \equiv 1 (mod\, 4)$, and is -1 if $p \equiv 3\, (mod\, 4)$.			
Q.3(a)	What are the simple continued fraction representations of 129 and $\sqrt{29}$?	[5]	CO3	
Q.3(b)	Find the least positive solution of $x^2 - 29y^2 = -1$ and $x^2 - 29y^2 = 1$, by generating the convergents of the expressions of $\sqrt{29}$.	[5]	CO3	
Q.4(a)	Find the remainder when $7^{1000000}$ is divided by 19 and 15! is divided by 19.	[5]	CO2	
Q.4(b)	Find the solutions for each of the individual linear relations given below.	[5]	CO2	
	(i) $6x \equiv 9 (mod\, 15)$ and			
	(ii) $6x \equiv 5 (mod\, 19)$.			
Q.5(a)	Find all integers n such that $\phi(n) = 2n$.	[5]	CO2	
	Find $\phi(1991)$.			
Q.5(b)	Check whether the following numbers are primes. Establish your claim	[5]	CO1	
	(i) 977 in the set of all integers.			
	(ii) $1 + \sqrt{-5}$ in $Z(\sqrt{-5})$.			

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