BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC | **SEMESTER: VII**
BRANCH: MATHEMATICS & COMPUTING | **SESSION: MO/2024**

SUBJECT: MA402 ADVANCED COMPLEX ANALYSIS

TIME: 3 Hours | **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

	Question	Marks	CO	BL
Q.1(a)	Prove that if a function $\boldsymbol{f}$ is entire and bounded, then $\boldsymbol{f}$ is constant throughout the plane.	[5]	2	1,2
Q.1(b)	Without actual doing the integration, obtain an upper bound on the absolute value of the integral $\int_C \frac{z+4}{z^3-1} dz$, where the integral is taken along the contour C, which is the arc of the circle $\lvert z\rvert = 2$ from $z = 2$ to $z = 2i$, lying in the first quadrant.	[5]	1	3
Q.2(a)	Develop the Laurent series expansions of the function $f(z) = \frac{z}{(z-1)(z-3)}$ in the regions: i) $1 < \lvert z\rvert < 3$ ii) $0 < \lvert z-1\rvert < 2$.	[5]	2	3
Q.2(b)	Prove that the real axis $y = 0$ is mapped onto the circle under the linear fractional transformation $w = \frac{iz+2}{4z+i}$. Find the center and radius of the image circle.	[5]	1	2
Q.3(a)	State and prove Residue theorem.	[5]	3	1,2
Q.3(b)	Identify whether the function $f(z) = (z^2-9)^{\frac{1}{2}}$ is a multivalued function or not. If it is so, obtain the branch point(s) and branch cut for it.	[5]	1	3
Q.4(a)	If a function $f(z)$ is meromorphic inside a simple closed contour C and $f(z)$ is analytic and has no zeros on C, then prove that: $$\frac{1}{2\pi i}\oint_C \frac{f'(z)}{f(z)} dz = N - P \qquad (1)$$ where N is the number of zeros and P is the number of poles lying inside C (a pole or zero of order m must be counted m times). Using the above expression (1), compute the value of integral $\oint_C \frac{f'(z)}{f(z)} dz$ with $f(z) = \frac{z^2-1}{(z^2+z)^2}$, where C is the circle $\lvert z\rvert = 3$.	[5]	3	2,3
Q.4(b)	State Rouche's theorem. Apply it to determine the number of roots of the polynomial $p(z) = z^5 - 4z^3 + z - 1$ that are lying interior to the unit circle $\lvert z\rvert = 1$.	[5]	2	1,3
Q.5(a)	The following information about an entire function $f(z)$ is given i) zeros of the function are at $z = n$, where $n \in N$ (set of natural numbers) and origin is the zero of multiplicity 3. ii) genus of the canonical product $P(z)$ associated with the function is one, whereas the genus of the function itself is two. Is it possible to construct $f(z)$ in Weierstrass Factorized form? If possible, then find the suitable form (s) of $f(z)$.	[5]	2	2,3
Q.5(b)	Explain order of an entire function. Hence, identify the order of the function $f(z) = e^z$.	[5]	2	1,3

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