BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSc | **SEMESTER : VII**
BRANCH: MATHEMATICS & COMPUTING | **SESSION : MO/2024**

SUBJECT: MA401 REAL ANALYSIS AND MEASURE THEORY

TIME: 3 Hours | **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	BL
Q.1(a)	Show that a monotone increasing function f on $[a,b]$ is of bounded variation on $[a,b]$.	[5]	CO1	BT1
Q.1(b)	Prove that a function f of bounded variation on $[a,b]$ can be expressed as the difference of two monotone increasing functions.	[5]	CO1	BT3
Q.2(a)	Consider the real valued function $f:[1,2] \to \mathbb{R}$ defined by $f(x) = \begin{cases} -1 & x \text{ is irrational} \\ 1 & x \text{ is rational} \end{cases}$ Show that f is not Riemann integrable.	[5]	CO2	BT1
Q.2(b)	Suppose the real valued function $f:[c,d] \to \mathbb{R}$ is continuous and $\alpha : [c,d] \to \mathbb{R}$ is monotone increasing. Prove that there exists a point $\zeta \in [c,d]$ such that $\int_c^d f\, d\alpha = f(\zeta)[\alpha(d) - \alpha(c)]$.	[5]	CO2	BT3
Q.3	Prove that the outer measure of an interval in the real line $\mathbb{R}$ equals the length of the interval. Use it to find the outer measure of the set $[0,1] \setminus \mathbb{Q}$.	[10]	CO3	BT3, BT1
Q.4	Given a bounded measurable function f on a measurable set E of finite measure. Show that f is Lebesgue integrable over E.	[10]	CO4	BT1
Q.5	Suppose $\{f_n\}$ is a sequence of non negative measurable functions on a measurable set E that converges pointwise on E to the function f. Suppose $f_n \le f$ on E for each $n \in \mathbb{N}$. Prove that $\lim_{n\to\infty} \int f_n = \int f$ over E.	[10]	CO5	BT3

::::::26/11/2024::::::E