

**BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI**  
(MID SEMESTER EXAMINATION MO/2024)

CLASS: IMSc  
BRANCH: MATHS & COMPUTING

SEMESTER: V  
SESSION : MO/2024

SUBJECT: MA315 FINANCIAL MATHEMATICS

TIME: 02 Hours

FULL MARKS: 25

**INSTRUCTIONS:**

1. The question paper contains 5 questions each of 5 marks and total 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates

		CO	BL
Q.1(a)	Given that the future value of 950 subject to continuous compounding will be 1,000 after half a year, find the interest rate.	[2]	1 2
Q.1(b)	Show that the future value $V(t)$ for a risk-free asset increases as one of the parameters $m$ , $t$ , $r$ or $P$ increases, the others remaining unchanged.	[3]	1 4
Q.2(a)	How long will it take to earn 100 in interest if 1,000,000 is deposited at 10% compounded continuously?	[2]	2 2
Q.2(b)	If no dividends are paid, then show that $1 + K(n, m) = (1 + K(n + 1)) (1 + K(n + 2)) \dots (1 + K(m))$ . If the one step returns are independent, then show that $1 + E(K(n, m)) = (1 + E(K(n + 1))) (1 + E(K(n + 2))) \dots (1 + E(K(m)))$ , where $E(X)$ denotes the expectation of a random variable $X$ .	[3]	2 3
Q.3(a)	Show that if $X' < X''$ , then $C^E(X') > C^E(X'')$ , where $C^E$ denote the price of a European call. Similarly, show that $P^E(X'') > P^E(X')$ , where $P^E$ denote the price of a European put.	[2]	3 4
Q.3(b)	Show that the prices of American put and call options with the same strike price $X$ and expiry time $T$ on a stock that pays no dividends satisfy $S(0) - X e^{-rT} \geq C^A - P^A \geq S(0) - X$ , where $C^A$ and $P^A$ denote the price of an American call and an American Put respectively.	[3]	3 3
Q.4(a)	Suppose that if $X' < X''$ . Then show that $C^A(X') - C^A(X'') < X'' - X'$ .	[2]	3 4
Q.4(b)	Suppose that 32, 28 and $x$ are the possible values of $S(2)$ , where $S(2)$ represents the price of stock at time 2 units. Find $x$ , if stock prices follow a binomial tree.	[3]	3 3
Q.5(a)	Consider a stock that pays no dividends and trades at 15.60 per share. At 2.83, European calls with a strike price of 15 and an exercise date of three months are trading on the stock. The interest rate is $r = 6.72\%$ , compounded continuously. What is the price of a European put with the same strike price and exercise date?	[2]	3 2
Q.5(b)	Let $S(0) = 75$ dollars and let $u = 0.2$ and $d = -0.1$ . Suppose that you can borrow money at 12%, but the rate for deposits is lower at 8%. Find the values of the replicating portfolios for a put and a call.	[3]	3 2

:25/09/2024:E