BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSc **SEMESTER: V**
BRANCH: MATHS AND COMPUTING **SESSION: MO/2024**

SUBJECT: MA315 FINANCIAL MATHEMATICS

TIME: 3 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
***4*. Before attempting the question paper, be sure that you have got the correct question paper.**
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	BL
Q.1(a)	Find the present value of 100000 to be received after 25 years assuming continuous compounding at 6%. What is the maximum loan amount you can secure if the interest rate is 18%, you can make payments of 10,000 at the end of each year, and you aim to fully repay the loan over 10 years?	[5]	1	2
Q.1(b)	(I) Define logarithmic return k (n, m) of a risky asset. Discuss the relation between logarithmic return k (n, m) and K (n, m). (II) Suppose that stock prices follow a binomial tree, the possible values of S (2) being 121, 110 and 100. Find u and d when S (0) = 100 dollars.	[5]	2	2
Q.2(a)	Prove that for a stock that pays no dividends if $C^E - P^E > S(0) - X\, e^{-rT}$ holds, then an arbitrage opportunity occurs in the market. A share of INFOSYS stock can be purchased at Rs 2, 500 today or at Rs 2, 850 six months from now. Which of these prices is the spot price, and which is the forward price?	[5]	3	3
Q.2(b)	Show that if X' < X'', then $P^E(X'') - P^E(X') < e^{-rT}(X'' - X')$, where P^E (X) is the price of a put option. Let B (0) = 100, B (1) = 120, S (0) = 100 and S (1) take values 120 and 80 with probabilities 0.8 and 0.2 respectively. Let C be a European call with the strike price X = 100 and T = 1 year. Find the replicating portfolio (x, y) for the call C.	[5]	3	2
Q.3(a)	What do you mean by American and European call option. Consider the Binomial Model with step size 1. Assume that the stock price at time 1 is increased by a factor (1+u), such that S (1) = S (0) (1+u). Show that the price of a call option grows with u, the other variables being kept constant.	[5]	4	3
Q.3(b)	(I) Suppose that S (0) = 17, F (0, 1) = 18, r = 8%, and short- selling requires a 30% security deposit attracting interest at d = 4%. Is there an arbitrage opportunity? Find the highest rate d for which there is no arbitrage opportunity. (II) Let A (0) = 100, A (1) = 112 and S (0) = 34. Is it possible to find an arbitrage opportunity if the forward price of stock is F = 38.60 with delivery date 1? (i.e.; T=1)	[5]	3	3
Q.4(a)	Prove that the forward price of a stock paying dividend div at time t, where 0 < t < T, is $F(0,T) = [S(0) - e^{-rt}\, div]\, e^{rT}$, where F (0, T) is the forward price, T is the expiry time.	[5]	3	2
Q.4(b)	Prove that for $-1 < \rho_{12} < 1$ the portfolio with minimum variance is attained at S=$\frac{\sigma_1^2 - \rho_{12}\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2}$, where ρ_{12} is the correlation between the security assets 1 and 2, σ_1 and σ_2 are the variances of the assets 1 and 2 respectively. Show that if $\rho = -1$ and $\sigma_1 \leq \sigma_2$, then the investor can eliminate the risk in the portfolio without restoring to short selling.	[5]	5	2

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Q.5(a)	Show that the variance of a portfolio, i.e. Var (K_V) is given by $\sigma_V^2 = w\,C\,w^T$, where w is the weight matrix and C is the covariance matrix. T denotes the transpose of a matrix. Compute the expected return and standard deviation of a portfolio consisting of three securities with weights w_1 = 40%, w_2 = −30%, w_3 = 70%, given that the securities have expected returns μ_1 = 8%, μ_2 = 7%, μ_3 = 5%, standard deviations σ_1 = 1.5, σ_2 = 0.5, σ_3 = 1.2 and correlations ρ_{12} = 0.3, ρ_{23} = 0.0, ρ_{31} = −0.2.	[5]	5	2
Q.5(b)	When a portfolio is said to efficient? Define the efficient frontier. Prove that the portfolio with the smallest variance in the attainable set has weights $w = \frac{u\,C^{-1}}{u\,C^{-1}u^T}$, where C is the covariance matrix and u are the matrix consisting of all 1's.	[5]	5	3

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