BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION MO/2024)

CLASS: IMSs **SEMESTER : V**
BRANCH: MATHEMATICS **SESSION : MO/2024**

SUBJECT: MA301 PROBABILITY AND STATISTICS

TIME: 02 Hours **FULL MARKS: 25**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 5 marks and total 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates

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Q.1(a)	Let X be a random variable having probability density function $$f(x) = \begin{cases} \frac{2}{5}\lvert x\rvert, & -2 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$ Derive the cumulative distribution function $F(x)$.	[2]		
Q.1(b)	A student attempting a Multiple Choice Question (MCQ) with 4 choices (of which one is correct) knows the correct answer with probability $\frac{3}{4}$. If the student does not know, then the student guesses a random choice. Given that a question was answered correctly, what is the conditional probability that the student knows the answer?	[3]		
Q.2(a)	A student is given a Multiple Choice Question (MCQ) with 10 questions, each question with four possible answers. Assume that the student randomly chooses answers to the questions. What is the probability that the student will get exactly 3 questions correct?	[2]		
Q.2(b)	Let X_1 and X_2 be two independent geometric random variables with the same parameter $p \in (0,1)$. Derive the probability distribution of the sum $X = X_1 + X_2$.	[3]		
Q.3(a)	Let X be a real random variable with mean μ and finite variance σ^2. Show that $E[(X-a)^2] > E[(X-\mu)^2] = \sigma^2$,for all real numbers a satisfying $a \neq \mu$.	[2]		
Q.3(b)	Let $p_i = P(X = i)$ and suppose that $p_1 + p_2 + p_3 = 1$. If $E[X] = 2$, what values of p_1, p_2, p_3 maximize $Var(X)$?	[3]		
Q.4(a)	Assume that the height in inches of all college cricket players of BIT Mesra is a normally distributed random variable with mean $\mu = 71$ and variance $\sigma^2 = 6.25$ (i.e., $\sigma = 2.5$). Find the probability that college cricket players are over 74 inches tall. Note that $\Phi(1.1) = 0.8643$, $\Phi(1.2) = 0.8849$, where $\Phi(x)$ is the distribution of a standard normal random variable.	[2]		
Q.4(b)	Let X be a Poisson random variable with mean $\lambda > 0$. Compute the mean (or expectation) of the random variable $X!$. Compute $E[\lambda^X]$. (Note: $n! = n \cdot (n-1) \cdots 3 \cdot 2 \cdot 1$).	[3]		
Q.5(a)	Let X and Y be two jointly continuous random variables with the following joint probability density function (PDF) $$f(x,y) = \begin{cases} x + cy, & 0 \le x \le 1, x + y \le 1 \\ 0, & \text{otherwise} \end{cases}$$ Find the constant c that makes $f(x,y)$ a valid joint PDF. Also, find $P\left(0 \le X \le \frac{1}{2}, 0 \le Y \le \frac{1}{2}\right)$.	[2]		
Q.5(b)	Find $P\left(0 < X < \frac{1}{2} \mid X = \frac{1}{2}\right)$ using the marginal density $g(x)$ and the conditional density $f(y\|x)$.	[3]		

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