

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC
BRANCH: MATHEMATICS

SEMESTER: V
SESSION: MO/2024

SUBJECT: MA301 PROBABILITY AND STATISTICS

TIME: 3 Hours

FULL MARKS: 50

INSTRUCTIONS:

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
 2. Attempt all questions.
 3. The missing data, if any, may be assumed suitably.
 4. Before attempting the question paper, be sure that you have got the correct question paper.
 5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.
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- Q.1(a) The density function of a random variable X is given by [5]

$$f(x) = \begin{cases} a + bx^2, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

If $E[X] = \frac{3}{5}$, find a and b .

- Q.1(b) Let X be a continuous random variable having the probability density function (PDF) [5]

$$f(x) = \begin{cases} \frac{x^2}{3}, & -1 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

Verify that $f(x)$ is a density function. Derive the cumulative distribution function $F(x)$ and find $P(0 < X \leq 1)$.

- Q.2(a) Suppose you roll two dice and observe two numbers X and Y . Find the ranges R_X and R_Y , [5]
and the probability mass functions (PMFs) of X and Y . Find $P(X = 2, Y = 6)$ and
 $P(X > 3 | Y = 2)$.

- Q.2(b) A group of 5 boys and 10 girls is lined up in random order that is, each of the 15! [5]
permutations is assumed to be equally likely.

- (i) What is the probability that the person in the 4th position is a boy?
- (ii) What is the probability that a particular boy is in the 3rd position?

- Q.3 Let X and Y be two jointly continuous random variables with the following joint [10]
probability density function (PDF)

$$f(x, y) = \begin{cases} 10xy^2, & 0 < x < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the marginal densities $g(x)$, $h(y)$ and the conditional densities $f(x|y)$, $f(y|x)$.
Using the conditional density $f(y|x)$, find $P\left(Y > \frac{1}{2} | X = \frac{1}{4}\right)$.

- Q.4(a) Find the expectation and variance of the Poisson Distribution. [5]

- Q.4(b) Consider a random sample x_1, x_2, \dots, x_n from a normal distribution $N(\mu, \sigma)$. Find the [5]
maximum likelihood estimators for μ and σ^2 .

- Q.5(a) An electrical firm manufactures light bulbs that have a length of life that is [5]
approximately normally distributed, with mean equal to 800 hours and a standard
deviation of 40 hours. Find the probability that a random sample of 16 bulbs will have
an average life of less than 775 hours. Given that $\Phi(0.85) = 0.8023$ and $\Phi(-0.55) =$
0.2912. (Hint: use Central Limit Theorem)

- Q.5(b) A manufacturer of a certain brand of rice cereal claims that the average saturated fat [5]
content does not exceed 1.5 grams per serving. State the null and alternative hypotheses
to be used in testing this claim and determine where the critical region is located.