BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: BTECH/IMSC PHYSICS **SEMESTER: I/ADD**
BRANCH: ALL **SESSION: MO/2024**

SUBJECT: MA24101 MA103 MATHEMATICS - I

TIME: 3 HOURS **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data handbook/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	BL
Q.1(a)	Check the following positive term series for convergence and divergence: $$\left(\frac{1}{2}\right)^2 x+\left(\frac{1.3}{2.4}\right)^2 x^2+\left(\frac{1.3.5}{2.4.6}\right)^2 x^3+\left(\frac{1.3.5.7}{2.4.6.8}\right)^2 x^4+\cdots\ldots\ldots..,x>0.$$	[5]	1	2
Q.1(b)	Check the following series for absolutely or conditionally convergence: $$x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots\ldots\ldots$$	[5]	1	2
Q.2(a)	Find the values of a, b for which the system of equations: $$x+ay+z=3;\ x+2y+2z=b;\ x+5y+3z=9,$$ will have (i) no solution, (ii) a unique solution, (iii) infinite number of solutions.	[5]	2	1
Q.2(b)	Find the eigenvalues and eigenvectors of the matrix: $$A=\begin{bmatrix}2&0&1\\0&2&0\\1&0&2\end{bmatrix}$$	[5]	2	1
Q.3(a)	If $u=\frac{(x^2+y^2)^n}{2n(2n-1)}+xf\left(\frac{y}{x}\right)+g\left(\frac{x}{y}\right)$, then using Euler's theorem, evaluate: $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$	[5]	3	2
Q.3(b)	Examine the following functions for extreme values: $f(x,y)=x^3-y^3-2xy+6$	[5]	3	2
Q.4(a)	Change the order of integration and hence evaluate the following double integral: $$\int_0^\infty\int_0^x x\,e^{(-x^2/y)}\,dy\,dx$$	[5]	4	2
Q.4(b)	Find the volume of the portion of the sphere $x^2+y^2+z^2=a^2$ lying inside the cylinder $x^2+y^2=a^2$.	[5]	4	1
Q.5(a)	Show that $\vec{F}=(2xy+z^3)\,\hat{\imath}+x^2\,\hat{\jmath}+3xz^2\hat{k}$ is a conservative force field. Find the scalar potential. Find also the work done in moving an object in this field from (1, -2,1) to (3,1,4).	[5]	5	2
Q.5(b)	By Gauss's Divergence theorem evaluate $\iint_S \vec{F}.\hat{n}\,dS$ where $\vec{F}=x^3\,\hat{\imath}+zx^2\hat{k}$ and S is the closed surface bounded by the planes $x=0$, $x=6$ and cylinder $y^2+z^2=a^2$.	[5]	5	2

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