BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION MO/2024)

CLASS: IMSc **SEMESTER : III**
BRANCH: MATHEMATICS AND COMPUTING **SESSION : MO/2024**

SUBJECT: MA202R1 ABSTRACT ALGEBRA

TIME:02 hours **FULL MARKS: 25**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 5 marks and total 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates

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Q.1(a)	If $a \equiv b \bmod n$ and $c \equiv d \bmod n$, then prove that $$ac \equiv bd \bmod n$$	[2]	1	
Q.1(b)	Examine if the relation ρ on the set Z is an equivalence relation, where $$\rho = \{(a,b) \in Z \times Z : 3a + 4b \text{ is divisible by } 7\}$$	[3]	1	
Q.2(a)	Find all the cyclic subgroups of the group $(Z_5, +)$.	[2]	1	
Q.2(b)	Prove that $(z, .)$ is a group, where '.' is defined as $$a.b = a + b + 1 \ , a, b, \in Z$$	[3]	1	
Q.3(a)	Find $[G:H]$, where $H = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}\}$ and $G = (Z_6, +)$.	[2]	2	
Q.3(b)	Let $G = (Z, +)$ and $\phi: G -> G$ is defined by $\phi(x) = -x$, $x \in Z$. Prove that ϕ is a homomorphism and determine $ker\ \phi$.	[3]	2	
Q.4	Prove that, if $\phi : (G, .) -> (G', *)$ be a homomorphism, then ϕ is injective iff $Ker\ \phi = \{e_{G'}\}$, where $e_{G'}$ is the identity element of $(G', *)$.	[5]	2	
Q.5	Find the conjugacy classes $cl(\rho_0)$, $cl(\rho_1)$ and $cl(\rho_2)$ in S_3. Here $$\rho_0 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \ \rho_1 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \rho_2 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$ Note that others elements of S_3 follow usual notations.	[5]	3	

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