

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSc
BRANCH: MATHEMATICS AND COMPUTING

SEMESTER : III
SESSION : MO/2024

SUBJECT: MA202R1 ABSTRACT ALGEBRA

TIME: 3 Hours

FULL MARKS: 50

INSTRUCTIONS:

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
 2. Attempt all questions.
 3. The missing data, if any, may be assumed suitably.
 4. Before attempting the question paper, be sure that you have got the correct question paper.
 5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.
-

		CO	BL
Q.1(a)	Show that 293 is a prime number.	[5]	1
Q.1(b)	Define the order of an element in a group. Find order of each elements of $(Z_6, +, \cdot)$.	[5]	1
Q.2(a)	Define normal subgroups. Prove that $H = \{e_G\}$ is a normal subgroup of (G, \cdot) .	[5]	2
Q.2(b)	Let $\phi: (G, \cdot) \rightarrow (G', \cdot)$ be homomorphism. Then prove that ϕ is one to one iff $\ker \phi = \{e_G\}$.	[5]	2
Q.3(a)	Define the centre of a group. Prove that the centre (H, \cdot) is a subgroup of group (G, \cdot) .	[5]	3
Q.3(b)	Write all abelian groups of order 216.	[5]	3
Q.4(a)	Let $S = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$. Examine if $(S, +, \cdot)$ is a field.	[5]	4
Q.4(b)	Define divisors of zero in a ring. What are the divisors of zero in $(Z_8, +, \cdot)$.	[5]	4
Q.5(a)	Let $D = M_2(R)$, $D' = (R, +, \cdot)$ and $\phi : D \rightarrow D'$ defined by $\phi \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = \det \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad \text{where } \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_2(R).$	[5]	5
	Prove that ϕ is a ring homomorphism.		
Q.5(b)	Find a gcd of the elements $3 + i, 5 + i$ in $\mathbb{Z}[i]$.	[5]	5

:21/11/2024:E