BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSc. | **SEMESTER : III**
BRANCH: MATHEMATICS & COMPUTING | **SESSION : MO/2024**

SUBJECT: SUBJECT: MA201R1 PARTIAL DIFFERENTIAL EQUATIONS

TIME: 3 Hours | **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	BL
Q.1(a)	Derive the Partial differential equations by eliminating arbitrary functions from the following equation: $z = f(xy) + g\left(\frac{x}{y}\right)$	[5]	1	2
Q.1(b)	Solve the following linear Partial differential equation: $(x^2\text{-}y^2\text{-}z^2)p + 2xyq = 2zx$	[5]	1	1
Q.2(a)	Using Charpit's method, find the complete integral of the following PDE: $(p+y)^2 + (q+x)^2 = 1$	[5]	2	1
Q.2(b)	Solve the following linear Partial differential equation: $(D^3 - 3DD'^2 - 2D'^3)z = cos(x+2y)$	[5]	2	1
Q.3(a)	Derive the solutions of vibrations of a string of finite length by the method of serparation of variables: $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}, \quad 0 < x < a\,, \;\; t > 0$	[5]	3	2
Q.3(b)	Solve the PDE: $\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}\;, \quad 0 < x < 1\,, \;\; t > 0$ subject to the conditions: BCs: $y(0,t) = y(1,t) = 0, \quad t \geq 0.$ ICs.: $y(x,0) = f(x) = sin\,(\pi x) + 3sin\,(2\,\pi x), \qquad 0 \leq\; x \leq 1$ ICs.: $y_t(x,0) = g(x) = sin\,(\pi x), \quad 0 \leq\; x \leq 1$	[5]	3	1
Q.4(a)	Derive the solution of heat conduction problem for a finite rod by the method of serparation of variables: $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < L\,, \;\; t > 0$	[5]	4	2
Q.4(b)	Solve the PDE: $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 6\,, \;\; t > 0$ subject to the conditions: BCs: $u_x(0,t) = 0, \quad u_x(6,t) = 0, \; t \geq 0$ ICs.: $u(x,0) = f(x) = e^{-x}, \qquad 0 \leq x \leq 6$	[5]	4	1
Q.5(a)	Derive the solution of Dirichlet Problem for a Rectangle by the method of serparation of variables: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 \leq\; x \leq a, \quad 0 \leq\; y \leq b$	[5]	5	2
Q.5(b)	Solve the PDE: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 \leq\; x \leq 1, \quad 0 \leq\; y \leq 1$ subject to the conditions: $u_x(0,\text{y}) = 0, \quad u_x\,(1,\text{y}) = 0\,, \quad 0 \leq y \leq 1$ $u_y(\text{x},0) = 4\cos{(\pi x)}\;, \qquad u_y(\text{x},1) = 0\,, \;\; 0 \leq x \leq 1$	[5]	5	1

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