

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI  
(MID SEMESTER EXAMINATION)

CLASS: IMSC  
BRANCH: MATHEMATICS & COMPUTING

SEMESTER : 1<sup>ST</sup>  
SESSION : MO/2024

SUBJECT: MA102 REAL ANALYSIS

TIME: 02 Hours

FULL MARKS: 25

INSTRUCTIONS:

1. The question paper contains 5 questions each of 5 marks and total 25 marks.
  2. Attempt all questions.
  3. The missing data, if any, may be assumed suitably.
  4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates
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		CO	BL
Q.1(a)	State Completeness property of $\mathbb{R}$ . Give one example.	[2] CO1	1
Q.1(b)	Let $A$ be a non-empty subset of $\mathbb{R}$ , bounded above and $B = \{-x : x \in A\}$ . Prove that the set $B$ is bounded below and $\inf B = -\sup A$ .	[3] CO1	2
Q.2(a)	Find the limit point/points of the set $S = \left\{ \frac{(-1)^m}{m} + \frac{1}{n} : m, n \in \mathbb{N} \right\}$ . Justify.	[2] CO1	2
Q.2(b)	Prove that the set $S = \{x \in \mathbb{R} : 2x^2 - 5x + 2 > 0\}$ is an open set.	[3] CO1	2
Q.3(a)	Prove that the sequence $a_n = \frac{1}{\sqrt{n+1} + \sqrt{n}}$ is monotonic.	[2] CO2	2
Q.3(b)	Prove that a monotone decreasing sequence, if bounded below, is convergent and converges to its greatest lower bound.	[3] CO2	3
Q.4(a)	Examine the convergence/divergence of the sequence $\left\{ \frac{n}{3^n} \right\}$ .	[2] CO2	2
Q.4(b)	Prove that every convergent sequence is a Cauchy sequence.	[3] CO2	3
Q.5(a)	State D'Alembert ratio test.	[2] CO3	1
Q.5(b)	Using the limit comparison test, check whether the series $\sum \left( \frac{1}{n^3 + 5n + 4} \right)$ is convergent or divergent.	[3] CO3	2

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