BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSc | **SEMESTER : I**
BRANCH: MATHEMATICS & COMPUTING | **SESSION : MO/2024**

SUBJECT: MA102 REAL ANALYSIS

TIME: 3 Hours | **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	BL
Q.1(a)	Define interior point, limit point, open set and closed set with each one example.	[5]	CO1	1
Q.1(b)	Find the supremum and infimum of the following two set: $A = \{x \in \mathbb{R}: x^2 < 1\}$, $B = \left\{\frac{1}{m} + \frac{1}{n} : m, n \in \mathbb{N}\right\}$	[5]	CO1	2
Q.2(a)	Examine that the sequence $a_n = \frac{n}{n^2+1}$ is monotonic, bounded and convergent.	[5]	CO2	3
Q.2(b)	Consider $\{u_n\}, \{v_n\}, \{w_n\}$ be three sequences of real numbers and there is a natural number m such that $u_n < v_n < w_n$, for all $n \geq m$. If $\lim_{n\to\infty} u_n = \lim_{n\to\infty} w_n = L$, then prove that $\{v_n\}$ is convergent and $\lim_{n\to\infty} v_n = L$.	[5]	CO2	3
Q.3(a)	Using Cauchy Integral test, show that the series $\sum \frac{1}{n^p}$ is convergent when $p > 1$ and diverges for $0 < p \leq 1$.	[5]	CO3	3
Q.3(b)	Test the convergence and divergence of the series: $$\frac{x}{1} + \frac{1}{2}.\frac{x^3}{3} + \left(\frac{1.3}{2.4}\right)\frac{x^5}{5} + \left(\frac{1.3.5}{2.4.6}\right)\frac{x^7}{7} + \cdots \infty, (x > 0)$$	[5]	CO3	2
Q.4(a)	For each $n \in N$, let $f_n(x) = \frac{x}{1+nx^2}$ $\forall\, x \in [0,1]$, then check using the Mn-test the sequence of function $\{f_n\}$ is uniformly convergence or not over $[0, 1]$.	[5]	CO4	2
Q.4(b)	Prove that the series of function $1 + x + x^2 + x^3 + \cdots, 0 \leq x < 1$ is pointwise convergent on $0 \leq x < 1$, but the convergence is not uniform on $[0,1)$. (Hint: Use sequence of partial sum.)	[5]	CO4	3
Q.5(a)	A function f is defined by $f(x) = x^2, x \in [0,10]$. Find the upper integral and lower integral on the partition $P = \{0, 2, 4, 6, 8, 10\}$. Is f integrable over $[0, 10]$?	[5]	CO5	3
Q.5(b)	Consider a function $f: [a, b] \to \mathbb{R}$ be bounded on $[a, b]$. Then prove that f is integrable on $[a, b]$ iff for each $\epsilon > 0$, there exists a partition P of $[a, b]$ such that $U(P, f) - L(P, F) < \epsilon$.	[5]	CO5	3

::::::10/12/2024::::::M