BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION)

CLASS: IMSC **SEMESTER : 1ST**
BRANCH: MATHS & COMP. **SESSION : MO/2024**

SUBJECT: MA101 CALCULUS - I

TIME: 02 Hours **FULL MARKS: 25**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 5 marks and total 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
***4*. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates**

			CO	BL
Q.1(a)	Use Taylor's theorem to approximate $f(x) = e^{-x^2}$ around x =0 up to the third-order term.	[2]	1	2
Q.1(b)	Determine the points at which the function $f(x) = x^4 - 4x^3 + 6x^2$ is increasing and where it is concave up or down.	[3]	1	2
Q.2(a)	For the function $f(x) = xsin\left(\frac{1}{x}\right), x \neq 0$, show that the Lagrange Mean Value Theorem is not applicable on [−1,1].	[2]	1	2
Q.2(b)	Using Leibniz's theorem find the fourth derivative of $f(x) = e^x$ and $g(x) = xsin(x)$	[3]	1	2
Q.3(a)	Define Rolle's Theorem and give its geometric interpretation.	[2]	2	1
Q.3(b)	In the two curves $C_1 : x = y^2$ and $C_2 : xy = k$, if they cut at right angles, find the value of k	[3]	2	2
Q.4(a)	Determine the asymptotes of the polar curve given by $r = \frac{a\theta}{\theta - 1}$	[2]	2	2
Q.4(b)	Find the radius of curvature for the curve $y = x^3 - 3x$ at the points where it touches the x-axis	[3]	2	2
Q.5(a)	Evaluate the existence of the limit for the function $f(x,y) = \frac{x(y-1)}{y(x-1)}$, as (x, y) → (0, 0)	[2]	3	2
Q.5(b)	Verify Euler's theorem for the function $u(x,y) = x^n sin\left(\frac{y}{x}\right)$	[3]	3	2

:::::21/10/2024 E:::::