

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC
BRANCH: MATHS & COMP.

SEMESTER : I
SESSION : MO/2024

SUBJECT: MA101 CALCULUS - I

TIME: 3 Hours

FULL MARKS: 50

INSTRUCTIONS:

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

| | | CO | BL |
|--------|--|-------|-----|
| Q.1(a) | Evaluate the integral $\int_0^{2a} x^{\frac{7}{2}} (2a - x)^{-\frac{1}{2}} dx$ | [5] 1 | 1.5 |
| Q.1(b) | Expand $\frac{\sin^{-1}x}{\sqrt{1-x^2}}$ in powers of x. | [5] 1 | 1.5 |
| Q.2(a) | Find all the asymptotes of the curve $(a + x)^2(b^2 + x^2) = x^2y^2$. | [5] 2 | 2 |
| Q.2(b) | Show that $\left(\frac{21a}{16}, \frac{21a}{16}\right)$ is the center of curvature for the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on the curve $x^3 + y^3 = 3axy$ | [5] 2 | 2 |
| Q.3(a) | Use the method of Lagrange's multipliers to find the extreme values of $f(x, y, z) = 2x + 3y + z$ subject to $x^2 + y^2 = 5$ and $x + z = 1$. | [5] 3 | 2 |
| Q.3(b) | Expand x^y in powers of $(x - 1)$ and $(y - 1)$ up to the third-degree terms. | [5] 3 | 2 |
| Q.4(a) | Using reduction formula, evaluate $\int_0^{\frac{\pi}{2}} \cos^4(3\theta) \sin^3(6\theta) d\theta$ | [5] 4 | 2 |
| Q.4(b) | By differentiating under the integral sign, find $\frac{dF}{d\alpha}$ if $F(\alpha) = \int_0^{\alpha} \tan^{-1}(x/\alpha) dx$ | [5] 4 | 2 |
| Q.5(a) | Determine the area enclosed by the curves $y = \sin(x)$, $y = \cos(x)$, and the y-axis | [5] 5 | 1.5 |
| Q.5(b) | The area enclosed by the curve $y = 3e^{x/3}$, the x-axis, and ordinates $x = -1$ and $x = 3$ is rotated 360° about the x-axis. Determine the volume generated. | [5] 5 | 1.5 |

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