Q.4(a)	Use arithmetic encoding to find the range of probabilities for transmitting the sequence 'IMAGE' from left to right. The probability of occurrence for each source symbol is given below.	[6]	4	III

Symbol	I	M	A	G	E
Probability	0.2	0.2	0.2	0.1	0.1

Q.4(b)	Describe the steps to encode an image using JPEG compression standard.	[4]	4	I
Q.5(a)	Define motion vector and explain motion estimation using a block diagram. Also, mention how motion estimation contributes to improved video compression.	[4]	5	I
Q.5(b)	Explain various steps involved in H.261 standard for video compression. Also, mention the drawbacks of H.261 standard and how they are eliminated by MPEG compression standard.	[6]	5	I

:::::22/11/2024 M:::::

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSc SEMESTER: V
BRANCH: QEDS SESSION: MO/2024

SUBJECT: ED307 PARAMTETRIC INFERENCE

TIME: 3 Hours FULL MARKS: 50

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data handbook/Graph paper etc. to be supplied to the candidates in the examination hall.

Q.1(a) Let $X_1, X_2, ..., X_n$ be a random sample of size n from a population having uniform distribution over the interval $\left(\frac{1}{3}, \theta\right)$, where $\theta > \frac{1}{3}$ is an unknown parameter. If $Y = \max\{X_1, X_2, ..., X_n\}$ then find an unbiased estimator of θ in terms of Y. [5]

Q.1(b) Let $X_1, X_2, ..., X_{10}$ be a random sample of size 10 from a population having $N(0, \theta^2)$ distribution, where $\theta > 0$ is an unknown parameter. Let $T = \frac{1}{10}\sum_{i=1}^{10} X_i^2$. If the mean square error of $cT\,(c > 0)$, as an estimator of θ^2, is minimized at $c = c_0$, then find the value of c_0. [5]

Q.2(a) Let $X_1, ..., X_n$ be a random sample of size $n(\geq 2)$ from a uniform distribution on the interval $[-\theta, \theta]$, where $\theta \in (0, \infty)$. Find a sufficient and a minimal sufficient statistic for θ. [5]

Q.2(b) Let $\left\{-1, -\frac{1}{2}, 1, \frac{5}{2}, 3\right\}$ be a realization of a random sample of size 5 from a population having $N\left(\frac{1}{2}, \sigma^2\right)$ distribution, where $\sigma > 0$ is an unknown parameter. Let T be an unbiased estimator of σ^2 whose variance is lowest among all unbiased estimators of σ^2. Then based on the above data, find the realized value of T. [5]

Q.3(a) Let X be a random variable with distribution Uniform (0, μ). Prior pdf of μ is $h(\mu)=\frac{3}{\mu^4}$, μ>1. Find posterior pdf of μ. If the absolute error loss function is used, then find posterior mean of μ when observed value of x is 2. [5]

Q.3(b) Let X be a random variable with Binomial (2, θ). Prior distribution of θ is Uniform (1/2, 1). Find posterior pdf of θ. If the squared error loss function is used, then find posterior mean of θ observed value of x is 1. [5]

Q.4(a) Let X_1, X_2, X_3 be a random sample from a Poisson distribution with mean $\lambda, \lambda > 0$. For testing $H_0 : \lambda = \frac{1}{8}$ against $H_1 : \lambda = 1$, a test rejects H_0 if and only if $X_1 + X_2 + X_3 > 1$. Find the size and power of this test. Also find probability of Type-I and Type -II Error. [5]

PTO

Q.4(b) Let $X_1,\ldots,X_n$ be a random sample from a population with the probability density function $f(x) = \frac{1}{2\theta}e^{-|x|/\theta}, x \in \mathbb{R}, \theta > 0$. Find the most powerful critical region for testing $H_\theta : \theta = 1$ against $H_1 : \theta = 2$. [5]

Q.5(a) Write Monotone likelihood ratio property. Give an example of distribution which satisfies this property. Write a statement of Karlin Rubin Theorem and justify with one example. [5]

Q.5(b) Let X_1, X_2 be a random sample from a distribution having a probability density function, [5]

$$f(x;\theta)\begin{cases} \frac{1}{\theta}e^{-\frac{x}{\theta}}, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases},$$

where $\theta \in (0,\infty)$ is an unknown parameter. For testing the null hypothesis $H_0 : \theta = 1$ against $H_1 : \theta \neq 1$, consider a test that rejects H_0 for small observed values of the statistic $W = \frac{X_1 + X_2}{2}$, that is reject H_0 if W≤ c for some constant c . If the observed values of X_1 and X_2 are 0.25 and 0.75, respectively, then find the $p-$value of this test.

::::::22/11/2024::::::M