BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION MO/2024)

CLASS: IMSc | **SEMESTER: V**
BRANCH: CQEDS | **SESSION: MO/2024**

SUBJECT: ED307 PARAMETRIC INFERENCE

TIME: 02 Hours | **FULL MARKS: 25**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 5 marks and total 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
***4*. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates**

			CO	BL
Q.1(a)	Let $X_1, X_2, X_3, \ldots, X_n$ be a random sample from N (μ, σ^2). Find mean and variance of sample mean and sample variance both.	[2]	1	3
Q.1(b)	Define asymptotic efficiency. Show that MLE of μ in a random sample $X_1, X_2, X_3, \ldots, X_n$ from N $(\mu, 1)$ is asymptotically efficient estimator of μ.	[3]	1	3
Q.2(a)	Let $X_1, X_2, X_3, \ldots, X_n$ be a random sample from N $(0, \sigma^2)$. Calculate Bias and Mean square error of MLE of σ^2.	[2]	2	4
Q.2(b)	Define sufficiency of a statistic. Using definition of sufficient statistics, prove that T= $X_1, +X_2$ is a sufficient estimator for a family of distribution Bernoulli (θ).	[3]	2	3
Q.3(a)	Let $X_1, X_2, X_3, \ldots, X_n$ be a random sample from N $(\mu, 1)$. Write Cramér-Rao lower bound for variance of an unbiased estimator of μ. Prove that sample mean is most efficient estimator of μ.	[2]	3	4
Q.3(b)	State Basu's theorem. Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from U (0, θ) distribution. Prove that nth order statistics $X_{(n)}$ is independent of ratio $\frac{X_{(1)}}{X_{(n)}}$.	[3]	3	4
Q.4(a)	Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from Poisson (θ) distribution. In the exponential family, derive the canonical form of the sufficient statistic for the parameter θ of a Poisson distribution.	[2]	4	5
Q.4(b)	Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from Poisson (θ) distribution. Find UMVUE of $e^{-\theta}$.	[3]	4	4
Q.5(a)	Define Ancillary statistics. Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from U (θ, $1 + \theta$) distribution. Find an ancillary statistics using this random sample.	[2]	5	3
Q.5(b)	Write statement of Neyman Factorization Theorem. Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from N (1, θ) distribution. Find sufficient estimator of θ using Neyman Factorization Theorem. Is this sufficient estimator also a minimal sufficient estimator?	[3]	5	4

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