BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSc | **SEMESTER : I**
BRANCH: QEDS | **SESSION: MO/2024**

SUBJECT: ED24101 INTRODUCTORY ANALYSIS

TIME: 03 Hours | **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates.
5. All the notations used in the question paper have usual meanings.

			CO	BL
Q.1(a)	Give an example of a set which is open and closed both. Justify your answer.	[4]	CO1	1
Q.1(b)	Using Cauchy's limit theorem show that the sequence $\left\{a_n^{\frac{1}{n}}\right\}$ where $a_n = \frac{n^n}{(n+1)(n+2)\ldots(n+n)}$ converges and find the limit.	[6]	CO1	2
Q.2(a)	Does the convergency of the sequence $\{a_n\}_{n=1}^{\infty}$ imply the convergency of the series $\sum_{n=1}^{\infty} a_n$? Justify your answer.	[3]	CO2	2
Q.2(b)	Show that the series $\frac{3..6.9\ldots3n}{7.10.13\ldots(3n+4)}x^n, x > 0$ converges for $x \leq 1$ and diverges for $x > 1$.	[5]	CO2	2
Q.3(a)	Find the n^{th} order derivative of $y = x^{n-1}\ln x$ at $x = \frac{1}{2}$.	[3]	CO3	3
Q.3(b)	If $y = \cos(m\sin^{-1}x)$, then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0$. Hence, determine $y_n(0)$.	[5+2]	CO3	3
Q.4(a)	Interpret Rolle's theorem geometrically, and verify the theorem for the function $f(x) = 1-(x-1)^{\frac{2}{3}}$ on $[0,2]$.	[2+3]	CO4	4
Q.4(b)	Apply L'Hôpital rule to find the values of a, b and c such that the function $f(x) = \begin{cases} \frac{x(a+b\cos x)-c\sin x}{x^5}, & if\ x \neq 0 \\ 1, & if\ x = 0. \end{cases}$ is continuous at $x = 0$.	[5]	CO4	4
Q.5	Providing all the necessary information, make a rough sketch of the curve $x^3 + y^3 + 6xy = 0$.	[10]	CO5	5

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