

**BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI**  
(MID SEMESTER EXAMINATION MO/2024)

CLASS: BTECH  
BRANCH: ECE

SEMESTER : III/ADD  
SESSION : MO/2024

SUBJECT: EC213 PROBABILITY AND RANDOM PROCESSES

TIME: 02 Hours

FULL MARKS: 25

**INSTRUCTIONS:**

1. The question paper contains 5 questions each of 5 marks and total 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates

		CO	BL
Q.1(a) Determine the probability of the card being either a red or a king when one card is drawn from a regular deck of 52 cards.	[2]	I	II
Q.1(b) A certain test for a particular cancer is known to be 95% accurate. A person submits to the test and the results are positive. Suppose that the person comes from a population of 100,000, where 2000 people suffer from that disease. What can be concluded about the probability that the person under test has that particular cancer.	[3]	I	V
Q.2(a) A fair dice is rolled. Consider the events $A = \{1,3,5\}$ , $B = \{1,2\}$ and $C = \{1,3,4,5\}$ . Find $P[(A \cup B)/C]$ .	[2]	I	II
Q.2(b) A box contains white and black balls. When two balls are drawn without replacement, suppose the probability that both are white is $1/3$ . (a) Find the smallest number of balls in the box. (b) How small can the total number of balls be if black balls are even in number?	[3]	I	V
Q.3(a) If $X$ is exponentially distributed with parameter $\lambda$ , find the value of $k$ such that $\frac{P(X > k)}{P(X \leq k)} = a$ .	[2]	II	I
Q.3(b) Determine the characteristic function, $\phi_X(\omega)$ of a random variable $X \sim N(\eta, \sigma^2)$ , From the $\phi_X(\omega)$ , find the first and second order moments.	[3]	II	V
Q.4(a) The density function $f_X(x)$ of random variable $X$ is defined as $f_X(x) = \begin{cases} \frac{k}{2} & \text{for } 0 < x \leq 2 \\ \frac{1}{4} & \text{for } 2 < x \leq 3 \end{cases}$ where $k$ is an unknown constant. Find and plot the distribution function, $F_X(x)$	[2]	II	II
Q.4(b) Define the Rayleigh distribution function and obtain its mean and variance.	[3]	II	I
Q.5(a) If joint density function of the random variables $X$ and $Y$ is given by $f_{XY}(x, y)$ , then prove that the marginal densities of $X$ and $Y$ will be $f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x, y) dy$ and $f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x, y) dx$ , respectively.	[2]	III	II
Q.5(b) The joint density function of the random variables $X$ and $Y$ is given by $f_{XY}(x, y) = \begin{cases} ke^{-(ax+by)} & \text{for } x > 0, y > 0 \\ 0 & \text{otherwise} \end{cases}$ , where $k$ is an unknown constant. Determine $P(X > Y)$ . Are $X$ and $Y$ independent?	[3]	III	V

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