

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: BTECH
BRANCH: ECE

SEMESTER : III/ADD
SESSION : MO/2024

SUBJECT: EC213 PROBABILITY & RANDOM PROCESS

TIME: 3 Hours

FULL MARKS: 50

INSTRUCTIONS:

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

		CO	BL
Q.1(a) Consider the identity $A(B \cup C) = AB \cup AC$ and show that if in a set identity all sets are replaced by their complements, all unions by intersections, and all intersections by unions, the identity is preserved.	[5]	1	I
Q.1(b) In a group of n people, prove that the probability that two or more persons will have the same birthday (month and date) is given by $1 - e^{-\frac{n(n-1)}{730}}$	[5]	1	V
Q.2(a) If X is a Gaussian random variable with zero mean, then prove that $Y = X^2$ represents a chi-square random variable with one degree of freedom.	[5]	2	V
Q.2(b) Prove that a Poisson distributed random variable with parameter λ has the equal value of mean and variance.	[5]	2	V
Q.3(a) If the function $f_{X,Y}(x,y) = \begin{cases} ae^{-2x} \cos\left(\frac{y}{2}\right) & \text{for } 0 \leq x \leq 1, 0 \leq y \leq \pi \\ 0 & \text{elsewhere} \end{cases}$, where a is a positive constant, is a valid joint density function. Find the constant a .	[5]	3	III
Q.3(b) Suppose X and Y are zero mean independent Gaussian random variables with common variance σ^2 . Define $r = \sqrt{X^2 + Y^2}$ and $\theta = \tan^{-1} \frac{Y}{X}$, where $ \theta < \pi$. Determine their joint density function $f_{r,\theta}(r,\theta)$. Are r and θ independent?	[5]	3	V
Q.4(a) State and prove the central limit theorem.	[5]	4	II
Q.4(b) A random variable X has a probability density function $f_X(x) = e^{-x}$ for $x \geq 0$. Using the Chebyshev inequality determine the probability, $P[X - 1 \geq 2]$. Further, find the actual probability, $P[X - 1 \geq 2]$.	[5]	4	V
Q.5(a) A random process is given by $X(t) = a \cos \omega t + b \sin \omega t$, where a and b are random variables. Prove that the process $X(t)$ is WSS, when the random variables a and b are uncorrelated and with equal variance.	[5]	5	V
Q.5(b) The autocorrelation, $R_{XX}(\tau)$ of the WSS process, $X(t)$ is given by $R_{XX}(\tau) = \begin{cases} 1 & \text{for } \tau \leq \frac{\alpha}{2} \\ 0 & \text{otherwise} \end{cases}$. Determine the power spectral density, $S(\omega)$ of the process $X(t)$.	[5]	5	V

:23/11/2024:E