BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

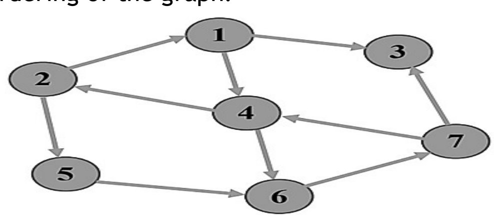
CLASS: IMSC | **SEMESTER: V**
BRANCH: MATHEMATICS AND COMPUTING | **SESSION: MO/2024**

SUBJECT: CS206 DESIGN AND ANALYSIS OF ALGORITHMS

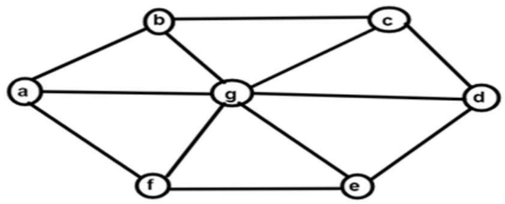
TIME: 3 Hours | **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
***4*. Before attempting the question paper, be sure that you have got the correct question paper.**
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	BL
Q.1(a)	Define the set $\Omega(f(n))$, where f(n) is an asymptotically non-negative function. Explain the significance of the Big Omega notation. Given $f(n) = 3n^2 + 2n + 2,\ g(n) = 4n^2 + 6$. Can we say that $f(n) = \Omega(g(n))$. Justify?	[5]	1	3
Q.1(b)	Draw the recurrence tree for the recurrence relation: $T(n) = T\left(\frac{n}{5}\right) + T\left(\frac{4n}{5}\right) + cn$. Solve the recurrence relation using recursion tree method	[5]	1	2
Q.2(a)	Convert the array A=<27, 17, 3, 16, 13, 10, 1, 5, 7, 12, 4, 8, 9> into a max-heap. Illustrate the operation HEAP-EXTRACT-MAX on the heap A=⟨15,13,9,5,12,8,7,4,0,6,2,1⟩	[5]	2	3
Q.2(b)	What is the worst-case running time of Quicksort? Identify the cases where the worst-case scenario occurs. What is the running time of QUICKSORT when all elements of the array have the same value? If the median element is selected as a pivot, what is the running time of Quicksort in that case.	[5]	2	2
Q.3(a)	Suppose a graph G is given in which each vertex is marked with a number as its index as shown below. Assume that if there is ever a choice amongst multiple nodes, the traversal algorithms will choose the node with lesser index first. Find the depth first tree and classify all the edges as tree edges, back edges, forward edges, and cross edges. Write the parenthesis structure of the traversal. Does there exists a topological ordering of the graph?	[5]	3	3



Q.3(b)	Compare Bellman Ford and Dijkstra's algorithm. Does Dijkstra's algorithm always give the shortest path from the source in presence of negative edge weights. Justify. Suppose that some edge costs in a directed graph G is negative. Make all edge costs positive by adding a fixed positive bias to each cost. Then run Dijkstra's algorithm on this updated graph. Give an example to demonstrate that this algorithm may fail to give the shortest paths in the original algorithm.	[5]	3	3
Q.4(a)	Consider the interval scheduling problem, where the objective is to find the maximum number of non-conflicting intervals. Identify the input and output of the problem. State the greedy strategy in order to find the maximum number of such intervals. Consider the following intervals: Interval 1: [1,4], Interval 2: [3,5], Interval 3: [0,6], Interval 4: [5,7], Interval 5: [8,9], Interval 6: [5,9]. Find the maximum number of such non-conflicting intervals.	[5]	4	2

Q.4(b)	The frequencies of the characters present in a symbol set are given as 0.19, 0.23, 0.03, 0.45, 0.05, and 0.05. Construct the Huffman's tree corresponding to the optimal prefix encoding. Find the average bits needed per character for the optimal prefix code. Compare the result with the fixed length encoding. You can choose the symbol set as {a, b, c, d, e, f}.	[5]	4	3
Q.5(a)	Define the class of problems P, NP and NP-Complete. A decision version of vertex cover problem is defined as follows: Given below a decision Vertex-cover problem instance (G, k) as given in the figure and k=4, find the equivalent set-cover instance 	[5]	5	4
Q.5(b)	Define the class of problems NP-hard. Consider the following Boolean 3-SAT formula $\varphi = (\neg x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee x_4)$. Convert the Boolean formula into an equivalent Independent Set instance.	[5]	5	4

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