BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

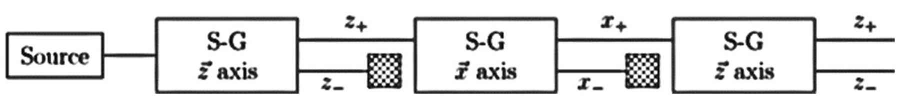
CLASS: I.M.Sc./M.Sc. **SEMESTER: VII/I**
BRANCH: PHYSICS **SESSION: MO/2023**

SUBJECT: PH404 QUANTUM MECHANICS - I

TIME: 3 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data handbook/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	**BL**
Q.1(a)	Show that the eigenvalues of a Hermitian operator are always real.	[3]	1	1,2
Q.1(b)	Consider a particle in the state $\lvert\psi\rangle = \frac{1}{2}\lvert a\rangle + \frac{\sqrt{3}}{2}\lvert b\rangle$. What is the probability of finding the particle in the states $\lvert a\rangle$ and $\lvert b\rangle$?	[2]	1	1,2
Q.1(c)	Show $(\sigma \cdot A)(\sigma \cdot B) = A \cdot B + i\,\sigma \cdot (A \times B)$ where σ are the Pauli matrices.	[3]	1	2
Q.1(d)	A one-dimensional ½-Quantum Harmonic Oscillator (½-QHO) is defined by the potential: $V(x) = \frac{1}{2}kx^2 \; \forall\, x > 0$, $V(x) = \infty \; \forall\, x \leq 0$. Starting from the known wavefunctions of a QHO and using symmetry arguments, plot the first two wavefunctions for a ½-QHO (expressions are not required).	[2]	2	3
Q.2(a)	Show that the ground state of a quantum harmonic oscillator is the least uncertainty state, *i.e.*, $\Delta x \Delta p = \frac{\hbar}{2}$.	[4]	1,2	1,2
Q.2(b)	Consider the following sequential Stern-Gerlach (S-G) setup consisting of S-G$\hat{z}$, followed by S-G$\hat{x}$ and another S-G$\hat{z}$, as shown in the Figure below.	[2]	1,3	1,5



	Determine the probability of finding the particle in $\lvert z+\rangle$ and $\lvert z-\rangle$ state (final output) if the middle Stern-Gerlach apparatus, S-G$\hat{x}$, is removed.			
Q.2(c)	Obtain the matrix representation for the position and momentum operators, $\hat{x}$ and $\hat{p}$, respectively, of a quantum harmonic oscillator in the position basis and show that $[\hat{x}, \hat{p}] \neq 0$.	[4]	1,2	1,2,4
Q.3(a)	Plot a comparison of the normalized radial probability distribution functions for the $1s$, $2s$, and $3s$ states of the hydrogen atom.	[3]	2	1
Q.3(b)	Let $J = J_1 + J_2$ be the total angular momentum of a system consisting of two particles with angular momenta J_2. Show that the operators J_1^2, J_2^2, J^2 and J_z commute with each other. Also show that while J^2 does not commute with J_{1z} and J_{2z} separately, it commutes with $(J_{1z} + J_{2z})$, thus establishing that the total angular momentum is a "good" quantum number.	[7]	1,3	1,2,5
	OR			
	Obtain the Clebsch-Gordon coefficients for $j_1 = \frac{1}{2}$ and $j_2 = \frac{1}{2}$.			

PTO

Q.4(a)	The particle exchange operator E operates on a two-particle state and interchanges the position of the particles. Using the principle of indistinguishability of identical particles, find the possible eigenvalues of E . Is E Hermitian or unitary? Explain.	[4]	1,3	1,5
	OR			
	Obtain the matrix representation of J^2 and J_z in their eigenbasis for $J = \frac{1}{2}$and show that they commute, *i.e.* $[J^2, J_z] = 0$.			
Q.4(b)	What are differential and total scattering cross-sections? Discuss the importance of partial wave analysis as applied to a spherically symmetric (scattering) potential.	[6]	4	1
Q.5	Discuss the phenomena of quantum tunnelling. How does the probability of finding the particle change with the width of the barrier.	[10]	5	1,3,5
	OR			
	Using the trial wavefunctions $\psi_t(r) = N \exp\left(-\alpha\left(\frac{r}{a_0}\right)^2\right)$, where N is the normalization constant and a_0 is the Bohr radii (natural length scale of the problem), outline the central idea(s) of variational principle as applied to the Hydrogen atom problem. Obtain the upper bound on the ground state energy. What is the true ground state wavefunction and energy?			

:::::30/11/2023 E:::::