

**BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)**

CLASS: I.MSC
BRANCH: PHYSICS

SEMESTER : III
SESSION : MO/2023

SUBJECT: PH301 QUANTUM MECHANICS AND APPLICATIONS

TIME: 3 HOURS

FULL MARKS: 50

INSTRUCTIONS:

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

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|--|-----|----|----|
| Q.1(a) What do you understand by Wave-particle duality? Calculate the wavelength associated with an electron subjected to a potential difference of 1.25 kV? | [5] | 1 | 4 |
| Q.1(b) Calculate the expectation value of p^2 for the wave function
$\psi(x) = \left(\frac{2}{L}\right)^{1/2} \sin\left(\frac{\pi x}{L}\right)$ in the region $0 < x < L$ and $\psi(x) = 0$ for $ x > L$ | [5] | 1 | 4 |
| Q.2(a) Discuss the physical significance of momentum wave function
$\phi_p(r) = \phi(k) = A e^{ik \cdot r}$ | [5] | 2 | 2 |
| Q.2(b) For a Gaussian function $f(x)$ represented by $f(x) = \frac{1}{\sqrt{(\sigma\sqrt{\pi})}} e^{-x^2/2\sigma^2}$ where σ determines the width of the wave-packet. Using Fourier transform derive the amplitude function $A(k)$? | [5] | 2 | 5 |
| Q.3(a) Derive the wavefunction and energy eigenvalues for a square well potential given by $V=0$ in the range $x=0$ to $x=L$, and $V=\infty$ elsewhere. | [5] | 3 | 2 |
| Q.3(b) Let $u(\xi)$ be the solution to the Hermite differential equation
$u''(\xi) - 2\xi u'(\xi) + (\Lambda - 1)u(\xi) = 0,$ then find the recurrence relation using the Frobenius method, $u(\xi) = \sum_r c_r \xi^{r+s}$. | [5] | 3 | 2 |
| Q.4(a) Using the known commutation relation between position and momentum operators, prove that $[L_x, L_y] = i\hbar L_z$, where the symbols have their usual meanings. | [5] | 4 | 3 |
| Q.4(b) The radial part of the wavefunction for the ground state of the hydrogen atom is given by $R_{10}(r) = (2/a_0)^{3/2} \exp(-r/a_0)$. Write down the full wavefunction $\psi_{100}(r, \theta, \phi)$. Show that the probability of finding the electron in the range $(r, r+dr)$ is proportional to $r^2 R_{10}^2(r)$ and is maximized at $r = a_0$. | [5] | 4 | 3 |
| Q.5(a) Discuss the Stern-Gerlach experiment and its significance? | [5] | 5 | 1 |
| Q.5(b) Write down the Pauli spin matrices. Show that the square of each Pauli spin matrix is given by the identity matrix, while the determinant of each equals -1. | [5] | 5 | 3 |

:::22/11/2023 M:::