BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC | **SEMESTER : IX**
BRANCH: MATHEMATICS & COMPUTING | **SESSION : MO/2023**

SUBJECT: MA501 FUNCTIONAL ANALYSIS

TIME: 3 HOURS | **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	**BL**
Q.1(a)	Let $l_p (p \geq 1)$ consist of all the sequences $x = \{x_n\}$ of complex numbers such that $\sum_{n=1}^{\infty} \|x_n\|^p < \infty$. Define d: $l_p \times l_p \to \mathbb{R}$ by $d(x, y) = (\sum_{n=1}^{\infty} \|x_n - y_n\|^p)^{1/p}, x = \{x_n\}$, y = $\{y_n\}$. Show that 'd' is a metric on l_p.	[5]	3	1
Q.1(b)	Show that a compact set in a metric space is closed and bounded. Does the converse hold?- Explain.	[5]	3	2
Q.2(a)	Define equivalent norms. On $\mathbb{R}^n$ for $x = (x_1, x_2, \ldots, x_n)$, define $\|\|x\|\|_1 = \sum_{i=1}^{n} \|x_i\|$ and $\|\|x\|\|_\infty = \max\{\|x_i\|, 1 \leq i \leq n\}$. Are $\|\| \; \|\|_1$ and $\|\| \; \|\|_\infty$ equivalent? Explain.	[5]	2,3	2
Q.2(b)	State Hahn-Banach theorem. Using it or otherwise show that first dual of a non-trivial normed space X is nonempty.	[5]	3	3
Q.3(a)	State and prove the closed graph theorem.	[5]	3	3
Q.3(b)	Show that the norm on an inner product space satisfies parallelogram laws and hence or otherwise show that the norm on l_p with $p \neq 2$ is not induced by an inner product.	[5]	3	3
Q.4(a)	Define orthogonal and orthonormal set in an inner product space. Show that a finite orthonormal set is linearly independent.	[5]	1	1,3
Q.4(b)	Consider C[−1, 1] with inner product defined as $< x, y >= \int_{-1}^{1} x(t)\bar{y}(t)dt$ and consider the set $\{t^3, t^2, t\}$. Orthonormalize it by Gram-Schmidt process in the said order.	[5]	1	3
Q.5(a)	Define adjoint and self-adjoint operators. Define an inner product on $\mathbb{C}^n$ by $< x, y > = x^T\bar{y}$. Let $T: \mathbb{C}^n \to \mathbb{C}^n$ be a self-adjoint operator. What will be the nature of the matrix representing T with respect to usual basis on $\mathbb{C}^n$? Explain.	[5]	4	1,2
Q.5(b)	Define an unitary operator. Show that a bounded linear operator T on a complex Hilbert space H is unitary if and only if T is isometric and surjective.	[5]	4	3

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