BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC/MSC | **SEMESTER : VII/IV**
BRANCH: MATHS. & COMPUTING/MATHEMATICS | **SESSION : MO/2023**

SUBJECT: MA402 ADVANCED COMPLEX ANALYSIS

TIME: 3 Hours | **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	BL
Q.1(a)	Discuss Cauchy-Goursat theorem for multiply connected domain. Hence, apply it to evaluate the integral $\oint_C \frac{1}{z^2+16}dz$, if $C:\lvert z\rvert = 5$ be positively oriented circle.	[5]	2	2,3
Q.1(b)	State and prove Liouville's theorem. Hence, use it to determine whether the function $f(z)=\cos z$ is a bounded function or not.	[5]	1	1,2
Q.2(a)	Derive that the linear fractional transformation $w = i\left(\frac{1-z}{1+z}\right)$ transforms the circle $\lvert z\rvert = 1$ onto the real axis of the $w-$plane and the interior of the circle $\lvert z\rvert < 1$ onto the upper half of the $w-$plane.	[5]	2	3
Q.2(b)	Explain analytic continuation. Describe how the following two series i. $\sum_{n=0}^{\infty}\frac{z^n}{2^{n+1}}$ ii. $\sum_{n=0}^{\infty}\frac{(z-i)^n}{(2-i)^{n+1}}$ are analytic continuations of each other.	[5]	1	2
Q.3(a)	Compute the residues at all the poles of the function $f(z)=\frac{1}{z(z+1)^2(z+3)}$. Hence, evaluate the integral $\oint_C f(z)dz$, where C is $\lvert z\rvert = 2$.	[5]	3	2
Q.3(b)	Prove that the function $f(z)=z^{1/2}$ is a two-valued function. Also, discuss about its branches, branch point and branch cut.	[5]	1	2
Q.4(a)	State Rouche's theorem. Hence, use it to determine the number of roots of the polynomial $p(z)=z^4+6z+10$ lying in the annular region $\frac{3}{2} < \lvert z\rvert < 2$.	[5]	3	1,3
Q.4(b)	If $f(z)$ is an analytic and non-constant function within and on a simple closed contour C, then prove that $f(z)$ attains its maximum value only on the boundary of C, and not inside C.	[5]	2	2
Q.5(a)	For an entire function $f(z)$ given in the form: $f(z)=z^4 e^{\sin z}\prod_{n=1}^{\infty}\left(1-\frac{z}{n}\right)e^{\frac{z}{n}+\frac{1}{2}\frac{z^2}{n^2}}$ i) Identify all the zeros of $f(z)$ along with their multiplicities. ii) Recognize the canonical product and its genus. iii) Obtain the rank and genus of function $f(z)$.	[5]	2	3
Q.5(b)	Define order of an entire function. Hence, apply the definition to identify the order of the function $f(z)=e^{z^5}$.	[5]	2	1,3

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