

**BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI**  
(END SEMESTER EXAMINATION MO 2023)

CLASS: IMSc  
BRANCH: MATH

SEMESTER : V  
SESSION : MO 2023

**SUBJECT: MA301 PROBABILITY AND STATISTICS**

TIME: 03 Hours

FULL MARKS: 50

**INSTRUCTIONS:**

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

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|--------|--|-----|---------------|------|
| Q.1(a) | State the Empirical definition of Probability as given by Von Mises. Mention one merit and one demerit of this definition.   | [2] | CO=1<br>Mod=1 | BL=1 |
| Q.1(b) | If two fair dice are thrown, find the chance that the sum is neither 7 nor 11.   | [3] | CO=1<br>Mod=1 | BL=2 |
| Q.1(c) | Find $E(X)$ for the distribution $P(X=x) = (1-p)^x p$ where $x=0, 1, 2, 3, \dots$ and $0 < p < 1$  | [5] | CO=1<br>Mod=1 | BL=3 |
| Q.2(a) | Give two real life examples in which Poisson Distribution will be a good model.  | [2] | CO=2<br>Mod=2 | BL=1 |
| Q.2(b) | Find $p$ for a Binomial variate given that $n=6$ and $9P(X=4) = P(X=2)$ . All symbols have usual meanings.   | [3] | CO=2<br>Mod=2 | BL=4 |
| Q.2(c) | Mention any two properties of normal distribution. Two independent random variates $X$ and $Y$ are both normally distributed with means 1 and 2 and standard deviations 3 and 4 respectively. If $Z = X - Y$ , find the distribution of $Z$ .  | [5] | CO=2<br>Mod=2 | BL=3 |
| Q.3(a) | Explain how a continuous random variable differs from a discrete random variable.  | [2] | CO=3<br>Mod=3 | BL=1 |
| Q.3(b) | A random variable has the pdf $f(x) = k, 2 < x < 3$ where $k$ is some constant. Find $k$ .   | [3] | CO=3<br>Mod=3 | BL=2 |
| Q.3(c) | The joint pdf of $X$ and $Y$ is $f(x,y) = x^3 y^3 / 16, 0 < x < 2, 0 < y < 2$ . Find the marginal densities of $X$ and $Y$ .   | [5] | CO=3<br>Mod=3 | BL=3 |
| Q.4(a) | When is an estimator called unbiased? Give an example of an unbiased estimator which is not consistent.  | [2] | CO=4<br>Mod=4 | BL=1 |
| Q.4(b) | For a random sample of size $n$ from Bernoulli distribution, prove that $(\sum x/n) (1 - \sum x/n)$ is a consistent estimator of $p(1-p)$ , where all symbols have usual meanings.   | [3] | CO=4<br>Mod=4 | BL=2 |
| Q.4(c) | State Factorization theorem and use it to find a sufficient estimator for the parameter $\phi$ of the distribution with pdf $f(x, \phi) = \phi x^{\phi-1}, 0 < x < 1, \phi > 0$  | [5] | CO=4<br>Mod=4 | BL=4 |
| Q.5(a) | Define the terms level of significance and power of the test.  | [2] | CO=5<br>Mod=5 | BL=1 |
| Q.5(b) | A die is thrown 64 times and the event $\{5, 6\}$ occurs 28 times. Is the die fair? Test at 5% level of significance.  | [3] | CO=5<br>Mod=5 | BL=3 |
| Q.5(c) | Let $p$ be the probability that a coin will fall Heads in a single toss. In order to test $H_0: p = \frac{1}{2}$ against $H_1: p = \frac{3}{4}$ the coin is tossed 5 times and the null hypothesis is rejected if more than 3 heads are obtained. Find the probabilities of type I and type II errors. | [5] | CO=5<br>Mod=5 | BL=4 |

:::24/11/2023:::M