

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION MO/2023)

CLASS: IMSc
BRANCH: MATHS & COMPUTING

SEMESTER : III
SESSION : MO/2023

SUBJECT: MA208 INTEGRAL TRANSFORM AND APPLICATIONS

TIME: 02 Hours

FULL MARKS: 25

INSTRUCTIONS:

1. The question paper contains 5 questions each of 5 marks and total 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates

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|--------|--|-----|------|------------|
| Q.1(a) | Find the Fourier Series of $f(x) = \begin{cases} \pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$. | [3] | CO-1 | BL
BT-1 |
| Q.1(b) | Hence deduce that $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots$ | [2] | CO-1 | BT-2 |
| Q.2 | Express the following function as Fourier half range cosine and sine series :
$f(x) = x + x^2, \quad 0 < x < 1$ | [5] | CO-1 | BT-2 |
| Q.3(a) | Solve the integral using Laplace Transform
$\int_0^\infty t e^{-2t} \cos 3t \, dt$ | [2] | CO-2 | BT-3 |
| Q.3(b) | Show that:
$L \left\{ \int_t^\infty \frac{\cos u}{u} du \right\} = \frac{1}{2s} \log(1 + s^2)$ | [3] | CO-2 | BT-3 |
| Q.4(a) | Find the following using convolution theorem:
$L^{-1} \left\{ \frac{1}{(s^2 + \omega^2)^2} \right\}$ | [2] | CO-2 | BT-1 |
| Q.4(b) | Evaluate:
$L^{-1} \left\{ \log \left(1 + \frac{1}{s^2} \right) \right\}$ | [3] | CO-2 | BT-3 |
| Q.5 | Using the Laplace transform technique, solve the following Initial value problem:
$y'' + 4y = t, \quad y(0) = 1, \quad y'(0) = 6$ | [5] | CO-2 | BT-3 |

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