**BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI**
**(MID SEMESTER EXAMINATION MO/2023)**

**CLASS: B.TECH.** **SEMESTER : III**
**BRANCH: MECH/CIVIL/CHEM/PIE/BIOTECH** **SESSION : MO/2023**

**SUBJECT: MA203 NUMERICAL METHODS**

**TIME: 02 Hours** **FULL MARKS: 25**

**INSTRUCTIONS:**
**1. The question paper contains 5 questions each of 5 marks and total 25 marks.**
**2. Attempt all questions.**
**3. The missing data, if any, may be assumed suitably.**
**4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates**

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| Q. | Question | Marks | CO | BL |
|---|---|---|---|---|
| Q.1(a) | Considering floating point arithmetic, perform the following arithmetic operations (as indicated) and express the obtained results in 4-digit mantissa standard (normalized) form with chopping:<br>i. $0.5466\times10^3+0.6115\times10^3$<br>ii. $0.8324\times10^5-0.8307\times10^5$ | [2] | 1 | 2 |
| Q.1(b) | The positive real root of the equation $f(x)=x^3-2x-5=0$ lies in the interval $(2,3)$. Taking $a=2$ and $b=3$ as the initial approximations, execute two iterations of Regula-Falsi method to get two approximate values of the root. At each iteration step, all calculations should be accurately done up to four decimal places. | [3] | 1 | 3 |
| Q.2(a) | State the convergence condition for the fixed point (general) iteration method $x=\phi(x)$. Hence, use it to check whether the following rearrangement of the fixed point (general) iteration method:<br>$x=x^3-0.1$<br>will ensure the convergence to the real root of the equation $f(x)=x^3-x-0.1$ in the interval $(1,2)$ or not. | [2] | 1 | 1,2 |
| Q.2(b) | Applying Newton-Raphson method, derive the following iterative formula to compute the cube root of any positive real number $N$ :<br>$x_{n+1}=\frac{1}{3}\left(2x_n+\frac{N}{x_n^{\ 2}}\right),\ n=0,1,2,....$<br>Hence, apply the above obtained iterative formula to find the cube root of $N=17$ correct to three decimal places taking $x_0=2.5$ as the initial approximation. | [3] | 1 | 2,3 |
| Q.3(a) | Let the matrix $\mathrm{A}=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. If $a\neq0$, the matrix $\mathrm{A}$ is to be factorized through $\mathrm{LU}$ decomposition where $\mathrm{L}=\begin{bmatrix} 1 & 0 \\ l_{21} & 1 \end{bmatrix}$ and $\mathrm{U}=\begin{bmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{bmatrix}$. Identify the values of $l_{21},u_{11},u_{12},$ and $u_{22}$ in terms of $a,b.c,$ and $d$. | [2] | 2 | 3 |
| Q.3(b) | Develop the solution of the following system of linear equations using Gauss elimination method:<br>$x+2y-3z=-2;x+5y+3z=10;x+3y+2z=5$<br>Clearly, mention the operations applied at each step. | [3] | 2 | 3 |

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| Q.4(a) | The system of linear equations is given as: $\begin{bmatrix} 5 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & -3 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$. Starting from an initial guess of $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, predict the result of the first iteration of Jacobi (Gauss-Jacobi) method. | [2] | 2 | 3 |
| Q.4(b) | The following system of linear equations is given: $$5x_1 - x_2 = 9; -x_1 + 5x_2 - x_3 = 4; -x_2 + 5x_3 = -6$$ Starting with initial approximations $x_1 = x_2 = x_3 = 0$, execute two iterations of Gauss-Seidel method to obtain approximate solution of the given system. | [3] | 2 | 3 |
| Q.5(a) | For the function $f(x) = \frac{1}{x^2}$, prove that the first divided difference $f[x_0, x_1]$ between the arguments (values of $x$) $x_0$ and $x_1$ is $$f[x_0, x_1] = -\left(\frac{x_0 + x_1}{x_0^2 x_1^2}\right).$$ | [2] | 3 | 2 |
| Q.5(b) | Obtain the interpolating polynomial that fits into the following data using Lagrange formula: | [3] | 3 | 3 |

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $f(x)$ | 2 | 1 | 12 |

**:::::19/09/2023 E:::::**