

**BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI**  
**(MID SEMESTER EXAMINATION MO/2023)**

**CLASS:** IMSc  
**BRANCH:** MATHEMATICS & COMPUTING

**SEMESTER :** I  
**SESSION :** MO/2023

**SUBJECT: MA102 REAL ANALYSIS**

**TIME:** 02 Hours

**FULL MARKS:** 25

**INSTRUCTIONS:**

1. The question paper contains 5 questions each of 5 marks and total 25 marks.
  2. Attempt all questions.
  3. The missing data, if any, may be assumed suitably.
  4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates
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		CO	BL
Q.1(a)	Define supremum and infimum of a non-empty subset of $\mathbb{R}$ .	[2]	1 1
Q.1(b)	Calculate the supremum and infimum of the set $\left\{1 + \frac{(-1)^n}{n}; n \in \mathbb{N}\right\}$ .	[3]	2 3
Q.2(a)	Explain if $\mathbb{N}$ , the set of all natural numbers is (i) an open set; (ii) a closed set; (iii) both open and closed set; (iv) neither an open nor a closed set.	[2]	2 2
Q.2(b)	Let $G \subset \mathbb{R}$ be an open set and $F \subset \mathbb{R}$ be a closed set. Then explain the nature of the set $G \cap F^c$ .	[3]	2 2
Q.3(a)	Find the formula for the nth term of the sequence $1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}, \dots$	[2]	1 1
Q.3(b)	Show that every convergent sequence is bounded. Does the converse hold?	[3]	2 3
Q.4(a)	Examine the monotonicity, boundedness and convergence of the sequence $x_n = \frac{(2n+3)!}{(n+1)!}$ ; $n=1, 2, 3, \dots$	[2]	1, 3 4
Q.4(b)	Find the limit of the sequence whose nth term is $\frac{\cos^2(n)}{n^2+1}$ .	[3]	3 3
Q.5(a)	Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$ .	[2]	3 3
Q.5(b)	Examine the convergence and divergence of the series $\sum_{n=1}^{\infty} \frac{n}{(2n-1)(2n+1)}$ .	[3]	3 4

**.....16/10/2023.....**