BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS:	**IMSC**	**SEMESTER : I**
BRANCH:	**MATHEMATICS & COMPUTING**	**SESSION : MO/2023**

SUBJECT: MA102 REAL ANALYSIS

TIME:	**3 HOURS**	**FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	**BL**
Q.1(a)	Compute the Supremum and Infimum of the set $\left\{\frac{1}{2^n} : n \in \mathbb{N}\right\}$.	[5]	1	3
Q.1(b)	Define an open set in $\mathbb{R}$. Show that finite intersection of open sets is open. Is it true infinitely many open sets?	[5]	1	1,3
Q.2(a)	Discuss the monotonicity, boundedness and convergence of the sequence (a_n) where $a_n = \frac{4^{n+1}+3^n}{4^n}, n = 1,2, \ldots.$	[5]	3	2
Q.2(b)	Define Cauchy sequence of real numbers and show that every Cauchy sequence is bounded	[5]	2	1,3
Q.3(a)	For the series $1 + \frac{1}{2}.\frac{1}{3} + \frac{1.3}{2.4}.\frac{1}{5} + \frac{1.3.5}{2.4.6}.\frac{1}{7} + \frac{1.3.5.7}{2.4.6.8}.\frac{1}{9}+\ldots.$ write the nth term and examine the convergence of the series.	[5]	3	4
Q.3(b)	Examine the convergence of the series $\sum_{n=1}^{\infty}(-1)^n \frac{n}{2^n+5}$.	[5]	3	4
Q.4(a)	Let (f_n) be a sequence of functions defined on $[0,\infty)$ by $f_n(x) = x^2 e^{-nx}$, $x \in [0,\infty)$. Find a function f such that (f_n) converges pointwise to f and further examine whether the convergence is uniform or not.	[5]	4	1,4
Q.4(b)	Show that the following series is uniformly convergent for all real x: $\sum_{n=1}^{\infty} \frac{1}{(n+x^2)^2}$.	[5]	4	3
Q.5(a)	A function f is defined on $[0,1]$ by $f(x) = x$, if x is rational; $= 0$, if x is irrational. Examine whether f is Riemann integrable on $[0,1]$.	[5]	5	4
Q.5(b)	A function f is defined on $[-3,3]$ by $f(x) = 2x Sin\frac{\pi}{x} - \pi Cos\frac{\pi}{x}, x \neq 0$; $= 0, x = 0$. Show that f is continuous on $[-3,3]$. Find an anti-derivative φ of f on $[-3,3]$ and hence or otherwise compute $\int_{-3}^{3} f(x)dx$.	[5]	4	3

:::::12/12/2023 M:::::