

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION MO/2023)

CLASS: IMSC
BRANCH: MATHS & COMP.

SEMESTER : I
SESSION : MO/2023

SUBJECT: MA101 CALCULUS - I

TIME: 02 Hours

FULL MARKS: 25

INSTRUCTIONS:

1. The question paper contains 5 questions each of 5 marks and total 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates

		CO	BL
Q.1(a)	Find the values of x for which the curve $y = x^3 - 9x^2 + 10x + 5$ is concave upward and downward	[2]	1 1
Q.1(b)	Find Taylor's series expansion of $f(x) = (2 + x)^{1/2}$ about the point 1	[3]	1 1
Q.2(a)	Find the n -th derivative of $y = e^{3x} \log x$	[2]	1 2
Q.2(b)	Verify the mean value theorem for the function $f(x) = \frac{ x }{x}$, $x \in [-1, 1]$	[3]	1 2
Q.3(a)	Find the radius of curvature at any point (x, y) on the rectangular hyperbola with equation $xy = c^2$	[2]	2 2
Q.3(b)	Find the asymptotes to the curve $y^2(x - 2a) = x^3 - a^3$	[3]	2 3
Q.4(a)	Show that $f(x, y) = \frac{xy}{\sqrt{x^2 + y^2}}$ when $(x, y) \neq (0, 0)$ $= 0$ when $(x, y) = (0, 0)$ is continuous at $(0, 0)$	[2]	3 2
Q.4(b)	Show that the circle of curvature at the origin of the parabola $mx + \frac{x^2}{a}$ is $x^2 + y^2 = a(1 + m^2)(y - mx)^3$	[3]	2 2
Q.5(a)	Check existence of limit of $\lim_{(x,y) \rightarrow (0,0)} \left(\frac{x^3 + y^3}{x - y} \right)$	[2]	3 1
Q.5(b)	If $f(x, y) = (x^2 + y^2) \tan^{-1} \frac{y}{x}$ when $x \neq 0$ and $f(0, y) = \frac{\pi y}{2}$, show using the definition of limits that $f_{xy}(0, 0) \neq f_{yx}(0, 0)$	[3]	2 2

:::13/10/2023:::