BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION MO/2023)

CLASS: Int. M.Sc. **SEMESTER : V**
BRANCH: QEDS **SESSION : MO/2023**

SUBJECT: ED307 PARAMETRIC INFERENCE

TIME: 03 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates

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Q.1(a)	Let $X_1, X_2, \dots, X_n$ be an i.i.d. random sample from a $Binomial(k, p)$ distribution where both k and p are unknown. Find the method of moments estimator of k and p.	[4]	CO1
Q.1(b)	Let $X_1, X_2, \dots, X_n$ be i.i.d. $Poisson(\theta), \theta \in [1, \infty)$. Find the Maximum Likelihood Estimator of θ.	[6]	CO2
Q.2(a)	Let $X_1, X_2, \dots, X_n$ be i.i.d. $N(\theta, \sigma^2)$ where both θ and σ^2 are unknown. Find the Maximum Likelihood Estimator of θ and σ^2. Are both estimators sufficient? Are they minimal sufficient?	[10]	CO5
Q.3(a)	Let $X_1, X_2, \dots, X_n$ be i.i.d. $Bernoulli(p)$. Show that the variance of $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ attains Cramer-Rao lower bound.	[4]	CO1
Q.3(b)	Define a conjugate family of distributions. Let $X \sim N(\theta, \sigma^2)$ and suppose the prior on θ is $X \sim N(\theta, t^2)$, where σ^2, μ, t^2 are all known. Then find the posterior distribution of θ and determine if Normal family of distributions is a conjugate family or not.	[6]	CO2
Q.4(a)	Let $X_1, X_2, \dots, X_n$ be a random sample from population having a pdf $f(x\|\theta) = e^{-(x-\theta)}, \quad \theta \leq x < \infty, \quad -\infty < \theta < \infty$. Construct a UMP($\alpha$) test for testing $H_0: \theta = 0$ against $H_a: \theta \neq 0$, where $\alpha \in (0,1)$.	[10]	CO3
Q.5(a)	Let $X_1, X_2, \dots, X_n$ be a random sample from $N(\mu, \sigma^2)$ population. Assuming that $\sigma = 1$ is known construct the UMP(0.05) test (critical region) for testing $H_0: \mu = 0$ against $H_a: \mu \neq 0$.	[5]	CO4
Q.5(b)	Let $X_1, X_2, \dots, X_n$ be a random sample from $N(\mu, \sigma^2)$ population. Assuming that σ is unknown construct the UMP(0.05) test (critical region) for testing $H_0: \mu = 0$ against $H_a: \mu \neq 0$.	[5]	CO4

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