BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION MO/2023)

CLASS: INT. M.Sc. | **SEMESTER : V**
BRANCH: QEDS | **SESSION : MO/2023**

SUBJECT: ED307 PARAMETRIC INFERENCE

TIME: 02 Hours | **FULL MARKS: 25**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 5 marks and total 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
***4*. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates**

			CO	BL
Q.1(a)	Let $X_1, X_2, \dots, X_n$ be a random sample from a distribution with the p.d.f. $$f(x\|\theta) = \frac{\theta}{x^2}, \ \ 0 < \theta < x < \infty$$ Then formulate the Maximum Likelihood Estimator and Method of Moments Estimator of θ.	[3]	CO2	
Q.1(b)	Find a sufficient statistic for θ using the same distribution given in 1(a).	[2]	CO1	
Q.2(a)	Let $X_1, X_2, \dots, X_n$ be i.i.d. $N(\theta, 1)$. One of the unbiased estimators of θ^2 is $\overline{X^2} - \left(\frac{1}{n}\right)$. Calculate its variance, and show that it is greater than the Cramer-Rao Lower Bound. [Hint: You may use Stein's identity $E[g(X)(X-\mu)] = \sigma^2 E[g'(X)]$, where μ is mean of X]	[5]	CO5	
Q.3(a)	Let $X_1, X_2, \dots, X_n$ be a random sample from a distribution with the p.d.f. $$f(x\|\theta) = \frac{1}{\theta}, \ \ x \in [0, \theta], \theta > 0$$ Then formulate the Maximum Likelihood Estimator and Method of Moments Estimator of θ.	[2]	CO2	
Q.3(b)	Compute the variances of both MLE and MM estimators. Give your reasons to decide which estimator you would prefer.	[3]	CO1	
Q.4(a)	Let $X_1, X_2, \dots, X_n$ be a random sample from a location exponential population with p.d.f. defined by $$f(x\|\theta) = e^{-(x-\theta)}, \quad \theta < x < \infty, \quad -\infty < \theta < \infty.$$ Then find a minimal sufficient statistic for θ.	[3]	CO1	
Q.4(b)	Let $X_1, X_2, \dots, X_n$ be a random sample from a $Gamma(\alpha, \beta)$ population. Then find a two dimensional jointly sufficient statistic for α and β. The p.d.f. of $Gamma(\alpha, \beta)$ is given as $f(x\|\alpha, \beta) = \frac{1}{\Gamma\alpha \, \beta^\alpha} x^{\alpha-1} e^{-x/\beta}, \ x > 0$.	[2]	CO1	
Q.5(a)	The random variable X takes the values 0,1,2 with the following distribution: $P(X=0) = p, \ P(X=1) = 3p, \ P(X=2) = 1-4p$, where $0 < p < \frac{1}{4}$. Determine whether the family of distributions of X is complete.	[3]	CO3	
Q.5(b)	The random variable X takes the values 0,1,2 with the following distribution: $P(X=0) = p, \ P(X=1) = p^2, \ P(X=2) = 1-p-p^2$, where $0 < p < \frac{1}{2}$. Determine whether the family of distributions of X is complete.	[2]	CO3	

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