BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION MO/2023)

CLASS: IMSc. | **SEMESTER : III**
BRANCH: QEDS | **SESSION : MO/2023**

SUBJECT: ED207 PROBABILITY II

TIME: 03 Hours | **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates

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CO

Q.1(a) Let X and Y be jointly continuous random variables with joint p.d.f. [5] CO1

$$f(x,y) = Kxy;\ 0 \le x \le 1, 0 \le y \le \sqrt{x}.$$

Then find the value of K and Find the marginal p.d.f. of X and Y. Determine whether X and Y are independent.

Q.1(b) Find the conditional p.d.f. of X given $Y = y$ for the p.d.f. given in 1(a). Also evaluate $E[X|Y = y]$ and $Var(X|Y = y)$. [5] CO1

Q.2 Let X and Y have a joint p.d.f. defined for $-\infty < x, y < \infty$ as [10] CO2

$$f(x,y) = \frac{5}{48\pi}\exp\left(-\frac{25}{32}\left(\frac{x^2}{4} - \frac{2x}{5} - \frac{xy}{5} + \frac{y^2}{9} - \frac{4y}{15} + \frac{4}{5}\right)\right)$$

Find the correlation coefficient of X and Y. Also find the means and variances of X and Y. What is the conditional p.d.f. of Y given $X = x$ and $E[Y|X = x]$? What is the conditional p.d.f. of X given $Y = y$ and $E[X|Y = y]$?

Q.3(a) A sample from a normal population produced variance 4.0. Find the size of the sample if the sample mean deviates from the population mean by no more than 2.0 with a probability of at least 0.95. [5] CO4

Q.3(b) A random sample of 5 is taken from a normal population with mean 2.5 and variance $\sigma^2 = 36$. Find the probability that the sample variance lies between 30 and 44. Find the probability that the sample mean lies between 1.3 and 3.5, while the sample variance lies between 30 and 44. (Write the probabilities in terms of CDF of sampling distribution) [5] CO4

Q.4 Let $\{X_n\}$ be a sequence of random variables with p.m.f. [10] CO5

$$f_n(x) = P(X_n = x) = \begin{cases} 1, & x = 2 + 1/n \\ x, & otherwise \end{cases}$$

Define the random variable X such that $X_n \to X$ in distribution. Does the sequence of distribution functions converge uniformly? Determine whether $X_n \to X$ in probability and almost surely.

Q.5(a) Let $\{X_n\}$ be a sequence of random variables with p.m.f. defined by $P(X_n = \pm(1/2^n)) = 1/2$, and 0 otherwise. Does the Lindeberg condition for CLT hold for this sequence? [4] CO3

Q.5(b) Let $X_1, X_2, \dots X_{100}$ be i.i.d. RVs with mean 75 and variance 225. Use Chebychev's inequality to calculate the probability that the sample mean will not differ from the population mean by more than 6. Then use the CLT to calculate the same probability and compare your results. [6] CO3

::::::24/11/2023::::::E