BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION MO/2023)

CLASS: INT. M.Sc. **SEMESTER : III**
BRANCH: QEDS **SESSION : MO/2023**

SUBJECT: ED207 PROBABILITY II

TIME: 02 Hours **FULL MARKS: 25**

INSTRUCTIONS:
1. **The question paper contains 5 questions each of 5 marks and total 25 marks.**
2. **Attempt all questions.**
3. **The missing data, if any, may be assumed suitably.**
4. **Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates**

			CO	BL
Q.1(a)	Let X and Y be jointly continuous random variables with joint p.d.f. $$f(x,y) = ce^{-(2x+3y)};\ \ x, y \geq 0.$$ Then find the value of c and determine whether X and Y are independent using marginal p.d.f. of X and Y.	[3]	CO1	
Q.1(b)	Evaluate $E[Y \mid X > 2]$ and $P(X > Y)$ for the p.d.f. given in 1(a).	[2]	CO1	
Q.2	Let $X_i, i = 1,2,3$ be independent with $N(i, i^2)$ distributions. Use the $X_i's$ to construct a statistic with (i) chi-squared distribution with 3 degrees of freedom (ii) t distribution with 2 degrees of freedom (iii) F distribution with 1 and 2 degrees of freedom.	[5]	CO1	
Q.3(a)	X and Y are independent random variables with $X \sim exponential(\lambda)$ and $Y \sim exponential(\mu)$. Let $Z = \min\{X, Y\}$ and $W = \begin{cases} 1, if\ \ Z = X \\ 0, if\ \ Z = Y \end{cases}$. Find the joint distribution of Z and W.	[3]	CO4	
Q.3(b)	Prove that Z and W are independent. (Hint: Show that $P(Z \leq z \mid W = i) = P(Z = z)$ for $i = 1,2$)	[2]	CO4	
Q.4	Let X and Y be standard normal random variables. Consider the transformation $U = X + Y$ and $V = X - Y$. Derive the joint density function of U and V. Also determine whether U and V are independent.	[5]	CO1	
Q.5(a)	Let X and Y have a joint p.d.f. defined for $-\infty < x, y < \infty$ as $$f(x,y) = \frac{1}{6\pi\sqrt{7}} \exp\left(-\frac{8}{7}\left(\frac{x^2}{16} - \frac{31x}{32} + \frac{xy}{8} + \frac{y^2}{9} - \frac{4y}{3} + \frac{71}{16}\right)\right)$$ Then find the means and variances of X and Y.	[3]	CO2	
Q.5(b)	Find the conditional p.d.f. of Y given $X = x$ and $E[Y \mid X = x]$.	[2]	CO2	

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