BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSc. **SEMESTER : I**
BRANCH: QEDS **SESSION : MO/2023**

SUBJECT: ED107 PROBABILITY - I

TIME: 3 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
***4*. Before attempting the question paper, be sure that you have got the correct question paper.**
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	BL
Q.1(a)	Five letters are addressed to five different people and the corresponding envelopes are prepared. The letters are put into the envelopes at random. What is the probability that no letter is in its proper envelope?	[5]	1	
Q.1(b)	There are two identical urns containing 4 white and 3 red balls: 3 white and 7 red balls. An urn is chosen at random, and a ball is drawn from it. Find the probability that the ball is white. What is the probability that it is from the first urn if the ball drawn is white?	[5]	1	
Q.2(a)	Evaluate the cumulative distribution function of the following distribution: $P[X=-1]=\frac{1}{7}$, $P[X=0]=\frac{2}{7}$, $P[X=2]=\frac{3}{7}$, $P[X=3]=\frac{1}{7}$.	[5]	2	
Q.2(b)	The probability that a screw manufactured by a machine to be defective is $\frac{1}{50}$. A lot of 6 screws are taken at random. Find the probability that (i) there are exactly 2 defective screws in the lot, (ii) no defective screw and (iii) at most 2 defective screws.	[5]	2	
Q.3(a)	Determine the value of the constant C such that f(x) defined by $f(x)=\begin{cases} C\,x\,(1-x), & 0<x<1 \\ 0, & otherwise. \end{cases}$ is a probability density function. Find the corresponding cumulative distributive function.	[5]	3	
Q.3(b)	Derive the Moment Generating Function of the above distribution.	[5]	4	
Q.4	Derive the Moment Generating Function of exponential distribution with rate parameter λ. Hence, obtain mean, variance and skewness measure.	[10]	4	
Q.5	The density function of a two-dimensional random variable (X,Y) is given by $f(x,y)=\begin{cases} kxy(x+y), 0\le x\le 1, 0\le y\le 1 \\ 0, \quad otherwise. \end{cases}$ Find (i) the value of k, (ii) the marginal density function of X and Y, (iii) $P\left[\frac{1}{2}\le X\le\frac{3}{4}, \frac{1}{3}\le Y\le\frac{2}{3}\right]$, (iv) $f_{X\mid Y}(x\mid y)$, $f_{Y\mid X}(y\mid x)$.	[10]	5	

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