BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION MO/2023)

CLASS:	**IMSC**	**SEMESTER : I**
BRANCH:	**QEDS**	**SESSION: MO/2023**

SUBJECT: ED101 INTRODUCTORY ANALYSIS

TIME:	**03 HOURS**	**FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates.
5. All the notations used in the question paper have usual meanings.

			CO	BL
Q.1(a)	Show that the set of integers $\mathbb{Z}$ is countable.	[4]	CO1	1
Q.1(b)	Test the convergency of the infinite series $\sum_{n=1}^{\infty}\frac{x^{2(n-1)}}{(n+1)\sqrt{n}}$, $x > 0$.	[6]	CO2	2
Q.2(a)	If $\{x_n\}$ be a sequence such that $x_{n+1} = 2 - \frac{1}{x_n}, n \geq 1$ and $x_1 = \frac{3}{2}$, then show that the sequence $\{x_n\}$ is bounded and monotonic, and that converges to 1.	[5]	CO2	2
Q.2(b)	Obtain the point(s) of discontinuity of the function f, defined on $\left[0, \frac{1}{2}\right]$ as $f(x) = \begin{cases} 0, x = 0 \\ \frac{1}{2} - x, 0 < x < \frac{1}{2} \\ \frac{1}{2}, x = \frac{1}{2} \end{cases}$. Also examine the kind of discontinuities.	[5]	CO3	2
Q.3(a)	Show that the function $f(x) = \sin x^2$ is not uniformly continuous on $[0, \infty)$.	[4]	CO3	3
Q.3(b)	Consider the function $f(x) = \begin{cases} x^3 \sin\frac{1}{x}, x \neq 0 \\ 0, x = 0 \end{cases}$. Check the continuity of the functions $f(x)$ and $f'(x)$ at $x = 0$.	[6]	CO4	3
Q.4(a)	Interpret Lagrange's mean value theorem geometrically, and verify the theorem for the function $f(x) = x(x-1)(x-2)$ on $\left[0, \frac{1}{2}\right]$.	[2+3]	CO4	4
Q.4(b)	Apply L'Hôpital rule to find the values of a, b and c such that $\lim_{x\to 0}\frac{x(a+b\cos x)-c\sin x}{x^5} = 1$.	[5]	CO4	4
Q.5	Providing all the necessary information, sketch the graph of the curve $y^2(a+x) = x^2(a-x)$.	[10]	CO5	5

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