## BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI (END SEMESTER EXAMINATION)

CLASS: BTECH SEMESTER: III SESSION: MO/2023

SUBJECT: EC213 PROBABILITY & RANDOM PROCESS

TIME: 3 Hours FULL MARKS: 50

## **INSTRUCTIONS:**

- 1. The question paper contains 5 questions each of 10 marks and total 50 marks.
- 2. Attempt all questions.
- 3. The missing data, if any, may be assumed suitably.
- 4. Before attempting the question paper, be sure that you have got the correct question paper.
- 5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

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Q.1(a)	A train and a bus arrive at the station at random between 9 A.M. and 10 A.M. The train stops for 10 minutes and the bus for $x$ minutes. Assuming the train and bus arrive independently, determine the value of $x$ so that the probability that the bus and the train will meet equals 0.5.	[5]	CO 1	BL V
Q.1(b)	A problem is given to 5 people A, B, C, D, E. If the probability of solving the problem individually is 1/6, 1/5, 2/3, 1/3, 1/2 respectively, then find the probability that the problem is solved.	[5]	1	I
Q.2(a)	Consider the experiment of tossing four coins. The random variable $X$ is associated with the number of trails showing. Compute and sketch the cumulative distribution function and probability density function of random variable $X$ .	[5]	2	Ш
Q.2(b)	Determine the 4th moment of the random variable $X \sim N(0, \sigma^2)$ .	[5]	2	٧
Q.3(a)	Let $f_{XY}(x,y) = \begin{cases} 1 & 0 <  y  < x < 1 \\ otherwise \end{cases}$ . Determine $E(X/Y)$ .	[5]	3	٧
Q.3(b)	Let $X$ and $Y$ be independent exponential random variables with common parameter $\lambda$ . Define $U=X+Y$ and $V=X-Y$ . Find the joint and marginal probability density functions of $U$ and $V$ .	[5]	3	I
Q.4(a) Q.4(b)	State and prove the Chernoff bounds. Suppose a fair coin is flipped 100 times. Find a bound on the probability that the number of times the coin lands is at least 60 or at most 40.	[5] [5]	4 4	I I
Q.5(a)	A stationary process $x(t)$ with autocorrelation $R_{xx}(t_1,t_2)=q\delta(t_1-t_2)$ is applied at $t=0$ to a linear system with impulse response $h(t)=e^{-at}U(t)$ . Determine the autocorrelation, $R_{yy}(t_1,t_2)$ of the resulting output $y(t)$ .	[5]	5	٧
Q.5(b)	Suppose that $X(t)$ is a process with mean, $\eta(t)=3t$ and autocorrelation, $R(t_1,t_2)=9+4e^{-0.2 t_1-t_2 }$ . Determine the variance and the covariance of the random variables $Z=X(5)$ and $W=X(8)$ .	[5]	5	٧

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