BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: MSC/IMSC **SEMESTER : III/IX**
BRANCH: PHYSICS **SESSION : MO/2022**

SUBJECT: PH504 NUMERICAL METHODS FOR PHYSICISTS

TIME: 3:00 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

Q.1(a) Describe the gradient descent algorithm and discuss the role of the learning rate. [2]

Q.1(b) Define the L_1-, L_2-, and L_∞-norms. What is the best representation of a dataset for each of these norms? [3]

Q.1(c) A company makes two products (X and Y) using two machines (A and B). Each unit of X that is produced requires 9 minutes processing time on machine A and 5 minutes processing time on machine B. Similarly, each unit of Y that is produced requires 20 minutes processing time on machine A and 8 minutes processing time on machine B. [5]

At the start of the week, there are 18 units of X and 30 units of Y in stock. Available processing times on machines A and B is forecast to be 6 hours and 5 hours, respectively.

The demand in the current week is forecast to be 25 units for X and 35 units for Y. Company policy is to maximize the combined sum of the units of X and units of Y in the stock at the end of the week.

- Formulate the problem of deciding how much of each product to make in the current week as a linear program.
- Solve the linear program graphically.

Q.2(a) Carry out the discrete Fourier transform for the following dataset: $X = [0, \sqrt{2}, 2, \sqrt{2}, 0, -\sqrt{2}, -2, -\sqrt{2}]$. Justify your result. [6]

Q.2(b) Carry out the fast Fourier transform for the following data set: $D = [0,1,1,0]$. Justify your result. [4]

Q.3(a) Discuss the properties of a matrix in a Jordan canonical form (including its definition). [2]

Q.3(b) Describe the QL decomposition algorithm for the eigenvalue problem. [3]

Q.3(c) Using power method, determine the dominant eigenvalue of the following matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$. Use $\lambda_1 = \frac{A^{2n+1}X_0}{A^{2n}X_0} \cdot \frac{A^{2n}X_0}{A^{2n}X_0}$ to check the convergence to the dominant eigenvalue. Explain your answer. [5]

Q.4(a) Illustrate the working of a pseudorandom number generator using linear congruence generator as an example. [2]

Q.4(b) Explain how Monte-Carlo simulations improve computational efficiency in large dimensions. In order to understand the properties of a magnetic material (solid), I wish to carry out a Monte Carlo study using a three-dimensional Heisenberg model and the Metropolis-Hastings algorithm. Provide the details of the sampling. [3]

Q.4(c) In a study from various streams of a college, 60 students, with equal number of students drawn from each stream, were interviewed and their intention to join the Drama club of college was noted. [5]

	B. Sc.	B. A.	B. Com.	M. A.	M. Com.
No. of students in each class	5	9	11	16	19

It was expected that 12 students from each class would join the Drama club. Using the KS (one sample) test, find if there is any difference among student classes with regard to their intention of joining the Drama club.

The tabulated value of D at 5% significance level is given by $D_{0.05} = \frac{1.36}{\sqrt{n}}$.

Q.5(a) Outline the distinction between finite difference method(s) and the finite element method. [2]

Q.5(b) Describe the classification of second order linear differential equations and their boundary conditions. [3]

Q.5(c) Provide the algorithm for solving the one-dimensional diffusion equation using finite difference method. Discuss the related subtleties in discretization. Outline the methods and highlight important methodological aspects to check the stability of the solution(s) obtained. [5]

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