BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC/MSC **SEMESTER : VII/I**
BRANCH: PHYSICS **SESSION : MO/2022**

SUBJECT: PH404 QUANTUM MECHANICS

TIME: 3:00 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

Q.1(a) Provide the outer product matrix |α⟩⟨α| and the inner product ⟨α|α⟩ of the vector [5]

$$|\alpha\rangle = \begin{pmatrix} 1 \\ 1.5 \\ 0.5 \end{pmatrix}$$

[CO1][BL2]

Q.1(b) Enlist the properties of a linear vector space with examples. [CO1][BL3] [5]

Q.2(a) If the annihilation and creation operators for a linear harmonic oscillator are defined as $a = (m\omega x + ip)/\sqrt{(2\hbar m\omega)}$ and $a^\dagger = (m\omega x - ip)/\sqrt{(2\hbar m\omega)}$, then use the commutation relation between position and momentum operators to show that $[a, a^\dagger] = 1$. [CO2][BL3] [5]

Q.2(b) The spherical harmonics denoted by $Y^m{}_l(\theta,\phi)$are eigenfunctions of the operator L^2, where L is the angular momentum operator. Explain why the parameter m can take integer values only. [CO2][BL2] [5]

Q.3(a) Explain the Stern-Gerlach experiment and its significance. [CO3][BL2] [5]

Q.3(b) Find the Clebsch-Gordan coefficients in the case of two particles with angular momenta j_1=j_2 =1/2. [CO3][BL3] [5]

Q.4(a) Provide a sketch of the classical scattering of a particle and explain the meanings of all the parameters involved. [CO4][BL2] [5]

Q.4(b) Explain why the differential cross-section $D(\theta)$ of a quantum particle is given in terms of the scattering amplitude $f(\theta)$ by the relation $D(\theta) = |f(\theta)|^2$. [CO4][BL2] [5]

Q.5(a) If E_ψ is the expectation value of a Hamiltonian with respect to an arbitrary state $|\psi>$, then show that it is always larger than the ground state energy E_0. [CO5][BL2] [5]

Q.5(b) Show that if the solution of a time-independent Schrodinger equation is given by $\psi(x) = A(x)\exp[i\phi(x)]$, then the position-dependent amplitude is given by $A(x) = C/\sqrt{\phi'(x)}$. Here, the prime denotes derivative with respect to x, and C is a constant. Assume that $E > V(x)$, where the symbols have their usual meanings. [CO5][BL2] [5]

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