

**BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)**

CLASS: MSC/IMSC
BRANCH: PHYSICS

SEMESTER : I / VII
SESSION : MO/2022

SUBJECT: PH401 MATHEMATICAL PHYSICS

TIME: 3:00 Hours

FULL MARKS: 50

INSTRUCTIONS:

1. The question paper contains 5 questions each of 10 marks and total 50 marks.
 2. Attempt all questions.
 3. The missing data, if any, may be assumed suitably.
 4. Before attempting the question paper, be sure that you have got the correct question paper.
 5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.
-

- Q.1(a) Explain when a function $f(z)$ is called an analytic function. [2]
 Q.1(b) Is the function: $u(x,y) = (y^3 - 3x^2y)$ harmonic? If yes, find its harmonic conjugate $v(x,y)$ and the function $f(z) = u(x, y) + i v(x, y)$. [3]
 Q.1(c) Evaluate the following integral: [5]

$$\int_0^{2\pi} \frac{d\theta}{(5 - 3\sin\theta)^2}$$

- Q.2(a) Use the generating function for the Legendre polynomials; [2]

$$g(t, x) = (1 - 2xt + t^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)t^n, \quad |t| < 1$$

- prove the recursion relation: $(2n + 1) x P_n(x) = (n + 1)P_{n+1}(x) + nP_{n-1}(x)$
 Q.2(b) Use the generating function for the Hermite polynomials; [3]

$$g(t, x) = e^{-t^2+2tx} = \sum_{n=0}^{\infty} H_n(x) \frac{t^n}{n!}$$

- Prove that:
 $H_n(x) = (-1)^n H_n(-x)$
 Q.2(c) Use the generating function for the Legendre polynomial, Prove that: [5]
 $|P_n(\cos\theta)| \leq P_n(1) = 1$

- Q.3(a) When two fair dice are thrown, Find the probability of: [2]
 (i) Observing a number < 4
 Observing a number ≥ 4 but < 6

- Q.3(b) Find the Laplace transform of $\int_0^t f(t)dt$. [3]

- Q.3(c) Find the Fourier transform of a triangular pulse. [5]

- Q.4(a) Show that the symmetric property of a tensor remains conserved under a transformation of cartesian coordinates. [2]

- Q.4(b) Discuss the quotient rule of the tensors. [3]

- Q.4(c) Compose a short note on Dyad. How its component can be calculated? [5]

- Q.5(a) Define an infinite group with an example. [2]

- Q.5(b) Compare two groups which are isomorphic to each other. Check whether they are abelian. [3]

- Q.5(c) Show that the symmetry elements of an equilateral triangle form a non-abelian group. [5]