BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

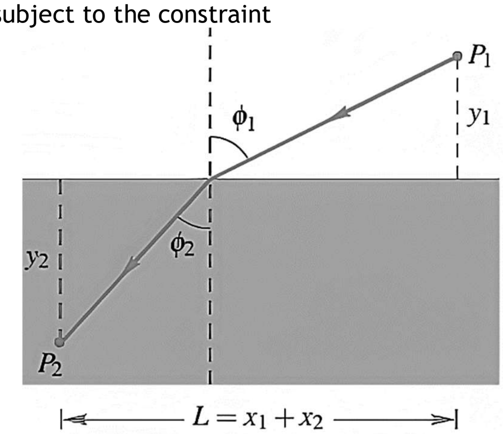
CLASS: IMSc. **SEMESTER: I**
BRANCH: PHYSICS **SESSION: MO/2022**

SUBJECT: PH105R1 MATHEMATICAL PHYSICS - I

TIME: 3 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data handbook/Graph paper etc. to be supplied to the candidates in the examination hall.

			CO	BL
Q.1(a)	Demonstrate whether the vectors $u = (1,0,1)$, $v = (1,1,0)$ and $w = (0,1,1)$ are linearly dependent or independent.	[2]	2	2
Q.1(b)	Prove that $A \cdot (A \times B) = 0$ using the properties of the Levi-Civita tensor.	[2]	2	2
Q.1(c)	Consider the first order differential equation: $x\frac{dy}{dx} + 3x + y = 0.$ After determining if this is exact or inexact, solve it.	[2]	1,3	5,6
Q.1(d)	A layered triangular lattice is spanned by the basis vectors (in real space): $a_1 = (a,0,0)$, $a_2 = \left(\frac{a}{2}, \frac{\sqrt{3}a}{2}, 0\right)$, $a_3 = (0,0,c)$. Determine the reciprocal set of vectors for such a lattice. Note that the resulting vectors form the basis set which spans the reciprocal space of a layered triangular/hexagonal lattice.	[4]	1,2	6
Q.2(a)	Compute the second order partial derivatives of $f(x,y) = x^3y - y^3x$.	[2]	1	3
Q.2(b)	Consider a simple harmonic oscillator (a spring mass system or a pendulum) of natural frequency ω_0 externally driven by a periodic force $f(t) = ma\cos(\omega t)$. The corresponding equation of motion is given by: $\frac{d^2x}{dt^2} + \omega_0^2 x = \frac{f(t)}{m}$, where x is the position. Given that at $t = 0$, we have $x = a$, $\frac{dx}{dt} = 0$, find $x(t)$. Discuss the solution if ω is approximately but not equal to ω_0.	[4]	1,3	6
Q.2(c)	In the case of inhomogeneous optical media consisting of two homogeneous media (M_1 and M_2) and their interface, the speed of light is piecewise constant in each homogeneous part (see Fig.). Suppose that light travels from point $P_1(x_1, y_1)$ with constant speed v_1 in medium M_1 to the point $P_2(x_2, y_2)$ with speed v_2 in medium M_2. By optimizing the time taken subject to the constraint	[4]	1,3	5,6



$L = x_1 + x_2$, show that $\frac{\sin\phi_1}{v_1} = \frac{\sin\phi_2}{v_2}$ (Snell's law).

Q.3(a)	Show that $\nabla r^n = nr^{n-2}\, r$.	[2]	1	2
Q.3(b)	Consider a particle moving on the perimeter of a circle, constrained due to a radial force. Obtain the direction of the force. What would be the equivalent expression if the particle is now constrained to move on the surface of a sphere.	[3]	3	
Q.3(c)	Consider a charge Q located at origin of the coordinate system. The electric field thereby produced will lead to flux through any surface (of your choice) enclosing the charge. Obtain the flux, defined as $\int_V \nabla \cdot E\, dV$, using Gauss's divergence theorem.	[5]	3,4	2,5
Q.4(a)	Verify Stoke's theorem for $A = (2x - y)\,\hat{\imath} - yz^2\,\hat{\jmath} - y^2 z\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.	[5]	4	2
Q.4(b)	Write down the properties of Dirac delta function and its representation using a box function, a Gaussian function and a Lorentzian function.	[5]	5	1,2
Q.5(a)	Derive the expression for ∇U in cylindrical coordinates.	[2]	3,5	1,2
Q.5(b)	Consider an arbitrary function $f(x,y)$ which can be re-expressed as $g(\rho,\phi)$, where ρ and ϕ are the plane polar coordinates. Transform the expression $\frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y}$ into one in ρ and ϕ .	[3]	3,5	2,6
Q.5(c)	Find the square of the element of arc length in spherical coordinate system and determine the corresponding scale factors. Subsequently, find the surface area element dS and the volume element dV .	[5]	5	2,5,6

:::::16/03/2023 M:::::