BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: MSC/PRE-PHD **SEMESTER : III/ I & II**
BRANCH: MATHEMATICS **SESSION : MO/2022**

SUBJECT: MA510 ADVANCED DIFFERENTIAL EQUATIONS

TIME: 3:00 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
***4*. Before attempting the question paper, be sure that you have got the correct question paper.**
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

Q.1(a) Derive that $f(x,y)=xy^2$ satisfies Lipschitz condition on the rectangle $R:|x|\le 1,\ |y|\le 1.$ [2]
(CO2, BT2)

Q.1(b) Identify whether the origin is an unstable or a stable equilibrium state for the following nonlinear system. [3]

$$\frac{dx}{dt}=y+3x^2;\frac{dy}{dt}=x-3y^2$$

(CO2, BT1)

Q.1(c) Construct the general solution of the following linear system: [5]

$$\frac{dx_1}{dt}=8x_1-4x_2;\frac{dx_2}{dt}=2x_1+2x_2$$

(CO2, BT3)

Q.2(a) The two first order partial differential equations are defined as: [2]

$$f(x,y,z,p,q)=0, g(x,y,z,p,q)=0$$

where, symbols have their usual meanings. Interpret the meaning of "compatible" partial differential equations in respect of the above partial differential equations. Also, state the mathematical condition under which the above partial differential equations form a compatible system. (CO3, BT2)

Q.2(b) Construct the complete integral of the following partial differential equation: [3]

$$q=3p^2$$

where symbols have their usual meanings. (CO3, BT3)

Q.2(c) Compute the solution of the following linear partial differential equation: [5]

$$\frac{\partial^2 z}{\partial x^2}-2\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial^2 z}{\partial y^2}=\sin x$$

(CO3, BT3)

Q.3(a) Using method of separation of variables with different possible values of separation constant, develop the general solution(s) of the one - dimensional wave equation which governs the vibrations of an elastic string of a finite length. (CO1, BT3) [5]

Q.3(b) Applying suitable transformations, determine the canonical form of the following one - dimensional wave equation that exhibits the vibrations of an infinite length elastic string: [5]

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}, x\in\Re, t>0$$

From the obtained canonical form, also deduce its general solution. (CO1, BT3)

Q.4(a) Classify the following partial differential equation as hyperbolic, parabolic or elliptic: [5]

$$\frac{\partial^2 z}{\partial x^2}+2\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial^2 z}{\partial y^2}=0$$

Determine the characteristics curves or lines of the above partial differential equation.
(CO4, BT3)

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Q.4(b) Using one - dimensional heat equation, obtain the temperature distribution in a thin metal rod of length 80 cm with its ends kept at 0° C and with the initial temperature distribution in the rod as $100\sin\left(\frac{\pi x}{80}\right)$. [5]

(CO4, BT3)

Q.5(a) With the application of method of separation of variables, solve the following Laplace equation: [5]

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

in a rectangle in the $x-y$ plane satisfying the boundary conditions: $u(x,0) = 0, u(x,b) = 0, u(0,y) = 0, u(a,y) = f(y)$

(CO5, BT3)

Q.5(b) Prove that if $u(x,y)$ is harmonic in some bounded domain D and continuous on D as well as on boundary B of D, then the maximum value of $u(x,y)$ occurs only on the boundary B of D. [5]

(CO5, BT2)

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