BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC/MSC **SEMESTER : IX / III**
BRANCH: MATHEMATICS **SESSION : MO/2022**

SUBJECT: MA507 COMPUTATIONAL FLUID DYNAMICS

TIME: 3:00 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
***4*. Before attempting the question paper, be sure that you have got the correct question paper.**
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

Q.1(a) Define and explain Dirichlet and Neumann boundary conditions with a suitable physical problem. [5]
Q.1(b) Determine the mathematical character of the equations given by [5]

$$\beta^2 \partial u/\partial x - \partial v/\partial y = 0$$

$$\partial v/\partial x - \partial u/\partial y = 0$$

Q.2(a) Derive the 2nd order central difference approximation of $\partial f/\partial x$ using Taylor series expansion. [5]
Q.2(b) The finite difference scheme of the linear convection equation $\partial u/\partial t + c\, \partial u/\partial x = 0$ is given by, [5]

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} = \frac{-c}{2\Delta x}\left(u_{j+1}^n - u_{j-1}^n\right)$$

Perform the Von Neumann stability analysis and comment on the stability of the scheme.

Q.3(a) What do you understand by explicit and implicit schemes? Explain with suitable examples. [5]
Q.3(b) Write down the modified Runge-Kutta method of order four to solve the inviscid Burgers equation $\partial u/\partial t + u\, \partial u/\partial x = 0$. [5]

Q.4(a) Derive the finite volume state update formula for Euler equations $\partial U/\partial t + \partial f/\partial x + \partial g/\partial y = 0$, where U=vector of conserved variables, f and g are flux vectors in x and y directions respectively. [5]
Q.4(b) Discuss briefly on global and local time steps. [5]

Q.5(a) Derive the pressure Poisson equation in primitive variables for solving incompressible flow. [5]
Q.5(b) What do you understand by staggered grid? Explain it for solving incompressible Navier-Stokes equations. [5]

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