BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: MSC/ IMSc **SEMESTER : III/IX**
BRANCH: MATHS /MATHEMATICS AND COMPUTING **SESSION : MO/2022**

SUBJECT: MA501 FUNCTIONAL AANALYSIS

TIME: 3:00 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

Q.1(a)	Let $C[a,b]$ denote the set of all real-valued continuous functions on $[a,b]$. For $x, y \epsilon C[a,b]$, define $d(x,y) = \int_a^b \mid x(t) - y(t) \mid dt$. Explain that 'd' is a metric on $C[a,b]$. BT2, CO3	[5]
Q.1(b)	Prove that if a closed unit ball becomes compact in a normed linear space X then X is finite-dimensional. BT2, CO3	[5]
Q.2(a)	Define a bounded linear operator over a normed linear space and give an example of a bounded linear operator. BT1, CO3	[2]
Q.2(b)	Prove that a linear operator defined over a finite-dimensional normed linear space is bounded. BT2, CO3	[3]
Q.2(c)	If X is a non-trivial normed linear space then show that the first dual of X is nonempty. BT3, CO3	[5]
Q.3(a)	If X and Y are two Banach spaces and $T:X \rightarrow Y$ is a closed linear operator, prove that T is bounded. BT2, CO3	[5]
Q.3(b)	Is the normed space X, of all polynomials with norm defined by $\|\|x\|\| = \max \|\alpha_j\|$ (α_0, α_1, ..., the coefficients of x) complete? Explain BT2, CO1	[5]
Q.4(a)	Show that every inner product space is a normed linear space. BT3, CO3	[2]
Q.4(b)	Let X be an inner product space. Suppose $x_n \rightarrow x$ and $y_n \rightarrow y$. Where will $\{\langle x_n, y_n \rangle\}$ converge? Interpret the result. BT2, CO3	[3]
Q.4(c)	State Riesz representation theorem of a bounded linear functional. Prove that every bounded linear functional f on l_2 can be represented by $f(x)=\Sigma x_i \bar{y}_i$ for $x=(x_i) \in l_2$ for specific $y=(y_i)$ BT3, CO3	[5]
Q.5(a)	Define Hilbert-adjoint and self-adjoint operators.. BT1, CO2	[2]
Q.5(b)	Let T: $l_2 \rightarrow l_2$ be defined by $T(\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, ...) = (0, \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, ...)$. Find the adjoint of T. BT2,CO2	[3]
Q.5(c)	Prove that a bounded linear operator T on a complex Hilbert space H is unitary if and only if T is isometric and surjective. BT2, CO2	[5]

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