

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI  
(MID SEMESTER EXAMINATION)

CLASS: IMSC.  
BRANCH: MATHS & COMPUTING

SEMESTER: V  
SESSION: MO/2022

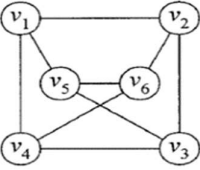
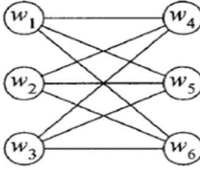
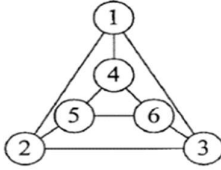
SUBJECT: MA305 GRAPH THEORY

TIME: 2 HOURS

FULL MARKS: 25

**INSTRUCTIONS:**

1. The total marks of the questions are 25.
  2. Candidates attempt for all 25 marks.
  3. Before attempting the question paper, be sure that you have got the correct question paper.
  4. The missing data, if any, may be assumed suitably.
  5. Tables/Data handbook/Graph paper etc. to be supplied to the candidates in the examination hall.
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- |        |   |     | CO | BL    |
|--------|---|-----|----|-------|
| Q1 (a) | State and prove Hand-Shaking lemma.   | [3] | 1  | 1     |
| Q1 (b) | What is the number of vertices in an undirected connected graph with 27 edges, 6 vertices of degree 2, 3 vertices of degree 4, and the remaining of degree 3?   | [2] | 4  | 3     |
| Q2 (a) | When do you say two graphs $G_1$ and $G_2$ are isomorphic graphs? Up to isomorphism, how many graphs with 3 vertices are there? Justify our answer.   | [2] | 1  | 1     |
| Q2 (b) | <div style="display: flex; justify-content: space-around; align-items: flex-end;"> <div style="text-align: center;">  <p><math>(G_1)</math></p> </div> <div style="text-align: center;">  <p><math>(G_2)</math></p> </div> <div style="text-align: center;">  <p><math>(G_3)</math></p> </div> </div> <p>Are the Graphs <math>G_1</math> and <math>G_2</math> isomorphic? Justify your answer.<br/>Are the Graphs <math>G_1</math> and <math>G_3</math> isomorphic? Justify your answer.</p> | [3] | 4  | 2,3   |
| Q3 (a) | Let $G$ be a simple graph with $n$ vertices and degree of each vertex is at least $\frac{n-1}{2}$ . Show that $G$ is connected.   | [3] | 1  | 2,3   |
| Q3 (b) | Is it possible to find a simple disconnected graph with $n$ vertices and degree of each vertex is at least $\frac{n-2}{2}$ ? Justify your answer.   | [2] | 1  | 2,3   |
| Q4 (a) | Let $G$ be a cycle on $n$ vertices. Show that $G$ is bipartite if and only if $n$ is even.  | [2] | 1  | 1     |
| Q4 (b) | Let $G$ be a cycle on $n$ vertices which is self-complementary. Show that the only value of $n$ is 5.   | [3] | 1  | 2,3   |
| Q5 (a) | Show that in any group of $n$ persons ( $n \geq 2$ ), there are at least two with same number of friends in the group.  | [2] | 4  | 3     |
| Q5 (b) | Let $G$ be a simple graph with $n \geq 3$ vertices. If for every pair of vertices $u, v$ of $G$ , $deg(u) + deg(v) \geq n - 1$ , then show that $G$ has a Hamiltonian path.   | [3] | 4  | 3,4,5 |