CLASS: IMSc.   SEMESTER : V
BRANCH: MATHS & COMPUTING   SESSION : MO/2022

SUBJECT: MA305 GRAPH THEORY

TIME: 3:00 Hours   FULL MARKS: 50

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

Q.1(a)   Define isomorphism between two graphs.   BT1 CO1   [2]
Q.1(b)   Let G_1 and G_2 be two isomorphic graphs. Show that the number of vertices and the number of edges   [3]
         in both the graphs are same.   BT3 CO1
Q.1(c)   [5]
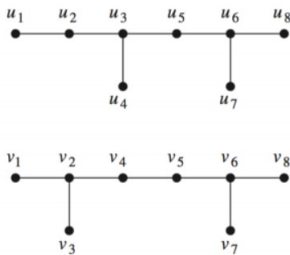
         Examine whether these two graphs are not isomorphic or not.   BT4 CO1

Q.2(a)   Show that in a connected Euler graph every vertex has even degree.   BT3 CO4   [2]
Q.2(b)   Which of the following graphs is a Euler graph?   [3]
         (i) Any k-regular graph where k is an even number
         (ii) A complete graph on 90 vertices
         (iii) The complement of a cycle on 25 vertices   BT4 CO4
Q.2(c)   Let G be a graph with 20 vertices having the property that for any two distinct vertices x, y of G,   [5]
         we have $deg(x) + deg(y) \geq 19$. Show that G is a connected graph.   BT3 CO4

Q.3(a)   Let G be a simple, connected and planar graph with $v \geq 3$ vertices and e edges. Show that   [2]
         $e \leq 3v - 6$.   BT3
         CO4
Q.3(b)   Let G be a simple, connected and planar graph with $v \geq 3$ vertices, e edges and no triangles. Show   3]
         that $e \leq 2v - 4$.   BT3 CO4
Q.3(c)   Prove that K_5 and $K_{3,3}$ are not planar graphs.   BT5 CO4   [5]

Q.4(a)   Define Chromatic number and Chromatic index of a graph.   BT1 CO3   [2]
Q.4(b)   [3]
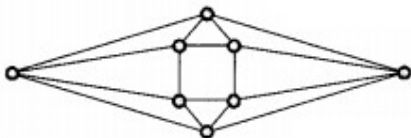

         Compute the Chromatic number and Chromatic index of the above graph.   BT3
         CO3
Q.4(c)   Compute the Chromatic polynomial of $K_4 - e$, where e is any edge of K_4.   [5]
         Show that there is no graph with Chromatic polynomial $k^5 - 4k^4 + 8k^3 - 4k^2 + k$. BT3 CO3

Q.5(a) Suppose a tree has even number of edges. Show that at least one vertex must have even degree. [2]
BT3 CO2

Q.5(b) Draw two non-isomorphic trees with same degree sequence. What is the smallest n for existence of [3]
such trees?   BT4 CO2

Q.5(c)   [5]
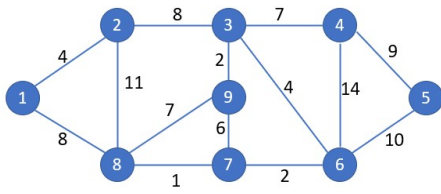
Use Prim's Algorithm to find a minimum spanning tree of the above graph.   BT3 CO5

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