

**BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION)**

CLASS: IMSC
BRANCH: MATHS & COMPUTING

SEMESTER: III
SESSION: MO/2022

SUBJECT: MA208 INTEGRAL TRANSFORM AND APPLICATIONS

TIME: 2 HOURS

FULL MARKS: 25

INSTRUCTIONS:

1. The total marks of the questions are 25.
2. Candidates attempt for all 25 marks.
3. Before attempting the question paper, be sure that you have got the correct question paper.
4. The missing data, if any, may be assumed suitably.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

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|--|-------|------|------------|
| Q1 (a) Find the Fourier Series of $f(x) = x $ in $-\pi < x < \pi$. | [3] | CO-1 | BL
BT-1 |
| Q1 (b) Hence using the above results, by Parseval's identity, express that
$\frac{\pi^4}{96} = 1 + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \dots$ | [2] | CO-1 | BT-2 |
| Q2 (a) Find the Fourier half range cosine Series of
$f(x) = x \sin x$ in $0 < x < \pi$. | [2] | CO-1 | BT-1 |
| Q2 (b) Hence, prove that: $\frac{\pi - 2}{4} = \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \dots$ | [3] | CO-1 | BT-5 |
| Q3 (a) Find the Laplace Transform of the function: $f(t) = te^{-4t} \sin 3t$ | [2.5] | CO-2 | BT-1 |
| Q3 (b) Find the Laplace Transform of the following full wave rectifier function:
$f(t) = \begin{cases} E \sin \omega t, & 0 < t < \lambda/\omega \\ 0, & \lambda/\omega < t < 2\lambda/\omega \end{cases}$
$f(t)$ is a periodic function of period $2\lambda/\omega$. | [2.5] | CO-2 | BT-1 |
| Q4 (a) Find the Inverse Laplace Transform of the function: $F(s) = \frac{se^{-2s}}{s^2 - 4}$ | [2] | CO-2 | BT-1 |
| Q4 (b) Using convolution theorem evaluate: $L^{-1}\left\{\frac{1}{(s-1)^5(s+2)}\right\}$ | [3] | CO-2 | BT-5 |
| Q5 (a) Solve the following partial differential equations with initial and boundary conditions using Laplace transform:
$x \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = xt,$
subject to the conditions : $u(x,0) = 0, u(0,t) = t$. | [5] | CO-2 | BT-3 |