BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: IMSC **SEMESTER: III**
BRANCH: MATHS & COMPUTING **SESSION: MO/2022**

SUBJECT: MA208 INTEGRAL TRANSFORM AND APPLICATIONS

TIME: 3 HOURS **FULL MARKS: 50**

INSTRUCTIONS:
1. The total marks of the questions are 50.
2. Candidates attempt for all 50 marks.
3. Before attempting the question paper, be sure that you have got the correct question paper.
4. The missing data, if any, may be assumed suitably.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

Q		Question	Marks	CO	BL
Q1	(a)	Find the Fourier Series of $f(x) = \sin(mx), \quad -\pi < x < \pi$, *where* m is not equal to zero and not an integer.	[5]	CO-1	BT-1
Q1	(b)	Find the Fourier Series representation of $f(x) = x - x^2, \quad -1 < x < 1$	[5]	CO-1	BT-1
Q2	(a)	Find the Laplace Transform of the function: $f(t) = te^{-2t} \cos 3t$	[2]	CO-1,2	BT-1
Q2	(b)	Using convolution theorem evaluate : $L^{-1}\left\{\frac{1}{(s^2+9)(s+2)}\right\}$	[3]	CO-1,2	BT-5
Q2	(c)	Solve the following initial value problem using Laplace transform: $y''(t) + 3y'(t) + 2y(t) = te^t, \; y(0) = 1, \; y'(0) = 0.$	[5]	CO-1,2,5	BT-3
Q3	(a)	Express the given function as a Fourier integral: $f(x) = \begin{cases} 1, & \lvert x\rvert < 1 \\ 0, & \lvert x\rvert > 1 \end{cases}$ Hence evaluate the following integral: $\int_0^\infty \frac{\sin(u)\sin(ux)}{u}\,du$	[5]	CO-1,3,5	BT-2,5
Q3	(b)	Find the Inverse Fourier Transform of the function : $F(s) = e^{-\lvert s\rvert y}$	[5]	CO-1,3	BT-1
Q4	(a)	If the Hankel Transform of order n of the function i.e., $H_n\{f(x)\} = F(s)$, then prove that: $H_n\{f(ax)\} = \frac{1}{a^2} F\left(\frac{s}{a}\right)$	[5]	CO-1,3	BT-5
Q4	(b)	Find the Inverse Hankel Transform: $H^{-1}\left[\frac{e^{-as}}{s}, \quad n = 0\right]$ Hence evaluate: $H^{-1}\left[\frac{1}{s}, \quad n = 0\right]$	[5]	CO-1,3	BT-5
Q5	(a)	If for casual sequence, $Z\{f(n)\} = F(z)$, Then prove that: $Z\{f(n+k)\} = z^k\left\{F(z) - f(0) - \frac{f(1)}{z} - \frac{f(2)}{z^2} - \cdots \ldots - \frac{f(k-1)}{z^{k-1}}\right\}$	[5]	CO-1,4,5	BT-5
Q5	(b)	Derive inverse Z transform if $F(z) = \frac{1}{(z-2)(z-3)}$ When (i) $\lvert z\rvert < 2$, (ii) $2 < \lvert z\rvert < 3$	[5]	CO-1,4,5	BT-4