BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION MO 2022)

CLASS:IMSc IMSC **SEMESTER : III**
BRANCH: MATHEMATICS AND COMPUTING **SESSION : MO/2022**

SUBJECT: MA201 PARTIAL DIFFERENTIAL EQUATIONS
TIME: 03 Hours **FULL MARKS: 50**

INSTRUCTIONS:
1. The question paper contains 5 questions each of 10 marks and total 50 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
***4*. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates**

Q.1(a) Find the partial differential equation arising from Z=ax+by+ab, where a and b are constants. CO1, BT2 [2]

Q.1(b) Obtain the partial differential equation by eliminating the arbitrary function f from z=f(x-y). CO2, BT2 [3]

Q.1(c) Find the general integral of yzp+xzq=xy, CO2, BT3 [5]

Q.2(a) Show that the integral surface of the equation $2y(u-3)p+(2x-u)q-y(2x-3)$ that passes through the circle $x^2+y^2=2x, u=0$ is $x^2+y^2-u^2-2x+4u=0$. CO3, BT3 [2]

Q.2(b) Solve the following Cauchy problem using method of characteristic $u_x+u_y=2$, $u(x,0)=x^2$ CO3,BT3 [3]

Q.2(c) Find a complete integral of $z=px+qy+p^2+q^2$ CO3, BT3 **[5]**

Q.3(a) Find a general solution of $(p+q)(z-xp-yq)=1$ CO3, BT2 [2]

Q.3(b) Solve $(D^2+3DD'+2D'^2)z=x+y$ CO3, BT3 [3]

Q.3(c) Reduce the following equations to canonical form and solve: [5]
$4u_{xx}-12u_{xy}+9u_{yy}=e^{3x+2y}$ CO2, BT3

Q.4(a) Classify the following partial differential equation $2u_{xx}+4u_{xy}+3u_{yy}=2$ CO1, BT1 [2]

Q.4(b) Show that the function $u=1/\sqrt{t}\ \exp(-x^2/4kt)$ satisfies one-dimensional heat equation. CO1, BT2 [3]

Q.4(c) Solve the following boundary value problem with following boundary conditions $u_t=u_{xx}$, $0<x<1$, $u(0,t)=0$, $u(1,t)=1$, $u(x,0)=x^2$ CO5, BT3 [5]

Q.5(a) Find D'Alembert's solution of one-dimensional wave equation with the following initial condition $u(x,0)=\sin x$, $u_t(x,0)=0$. CO1, BT2 [2]

Q.5(b) Using Duhamel's principle, solve $u_{tt}-u_{xx}=x-t$, $-\infty<x<\infty$, $u(x,0)=0$, $u_t(x,0)=0$ CO4, BT3 [3]

Q.5(c) Solve the following boundary value problem with the following boundary conditions $u_{xx}+u_{yy}=0$, $0<x<1$, $0<y<1$, $u(x,0)=x(x-1)$, $u(x,1)=0$, $0\le x\le 1$, $u(0,y)=0$, $u(1,y)=0$, $0\le y\le 1$. CO5, BT3 [5]

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